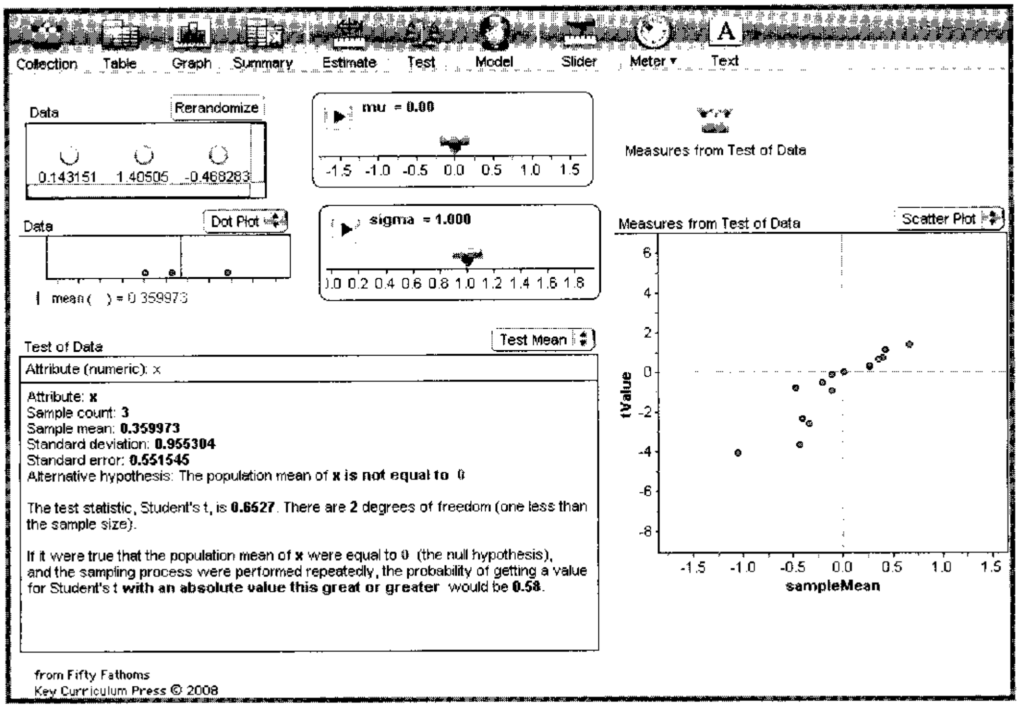
Demo 39: Another Look at a *t*-Test

Repeated t-tests on samples from the same distribution • How t, P, mean, and standard deviation interrelate

In this demo, we'll perform a *t*-test on the mean of a distribution that we have constructed ourselves. We'll know whether the null hypothesis is true. But then we'll change the distribution and see how the test changes, recording the results of these many *t*-tests. Throughout the demo, we'll look at those results as data to see what we can learn about the process.



What To Do

▹ Open **Exploring t.ftm.** It should look something like the illustration.

The specific numbers in your file will be different because of the randomization.

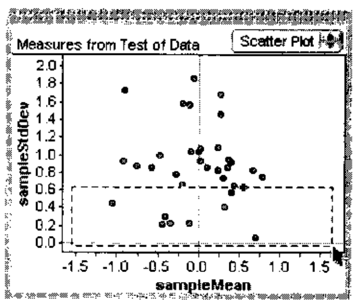
You can see the data in the upper left. Fathom draws the three values from a normal distribution controlled by the sliders **mu** and **sigma,** which determine the population mean and standard deviation, respectively. Below that is a graph of the data, and then a *t*-test, testing the null hypothesis that the mean is equal to zero. In the illustration, do you see how the sample mean is close to zero, *t* is small, and *p* is large? We cannot reject the null hypothesis, which is not a bad thing, especially since the null hypothesis is *true:* The simulation *sets* the population mean to zero!

▹ What else could have happened? Click the **Rerandomize** button a few times. Fathom redoes the *t*-test, and also plots the sample mean and the *t*-value in the graph at lower right.

You probably see a swath of points with a roughly positive slope. This makes sense because *t* is proportional to the sample mean (since we're testing against zero). But they aren't lined up perfectly, because *t* also depends on the sample standard deviation, which varies from run to run.

▹ Let's look at the standard deviation. Grab the lower-right corner of the test (the one with all the text) and drag it up and to the left until it turns into an icon. This reveals a graph of sample standard deviation versus sample mean.

▹ As shown in the illustration, drag a long horizontal rectangle in the new graph to select all the points with the smallest standard deviations.



▹ Notice where these points lie on the other graph; select other groups of points and figure out why they are where they are on each graph.

Note: There are at least two ways to "explain" such relationships. One is to relate them to the formula: Standard deviation is in the denominator for *t*, so a large SD means a small *t*. The other is to relate them to meaning: If a sample has a large SD, we're less sure about our estimate for the mean. So the larger the SD, the smaller the significance of any difference between the sample mean and what we're testing.

Questions

1 In the first graph, why are there points only in the first and third quadrants? Is it possible to have points anywhere in those quadrants? **Sol**

2 How can it be that one sample with a mean of 0.6 can have a smaller (that is, less significant) value for *t* than a different sample with a mean of 0.4? **Sol**

Onward!

Do you think one of these many *t*-tests rejects the null hypothesis? If so, that would be a Type I error, since the null hypothesis is true. Let's find out.

▹ On the right-hand graph, double-click the point with the largest (absolute) value for *t*. The collection's inspector opens to show that point. See the attribute's **pValue.** (Those attributes tell you the results of the test when it was performed; we have set up Fathom to collect these results automatically.)

Was it less than 0.05? Maybe, maybe not. Let's see some examples of tests where we would definitely reject the null hypothesis. A great way to do that is to make it false.

▹ Close the inspector.

▹ Drag the **mu** slider back and forth; Fathom collects points as fast as it can. You do not need to go any farther than ±1.5 to see everything.

▹ Now inspect the point with the most extreme *t*. Find its **pValue.**

▹ Let's look at **pValue** directly in the graphs. Drag its name from the inspector to the *middle* of each graph. It will turn colored, and you will see a spectrum-legend at the bottom.

More Questions

3 Why does the **pValue** have the range it does?

4 Look at the regions on each graph where **pValue** is near 1 (they're probably colored red). Why are those regions where they are?

5 Can you tell from a point on the graph whether or not it was made with **mu = 0**? **Sol**