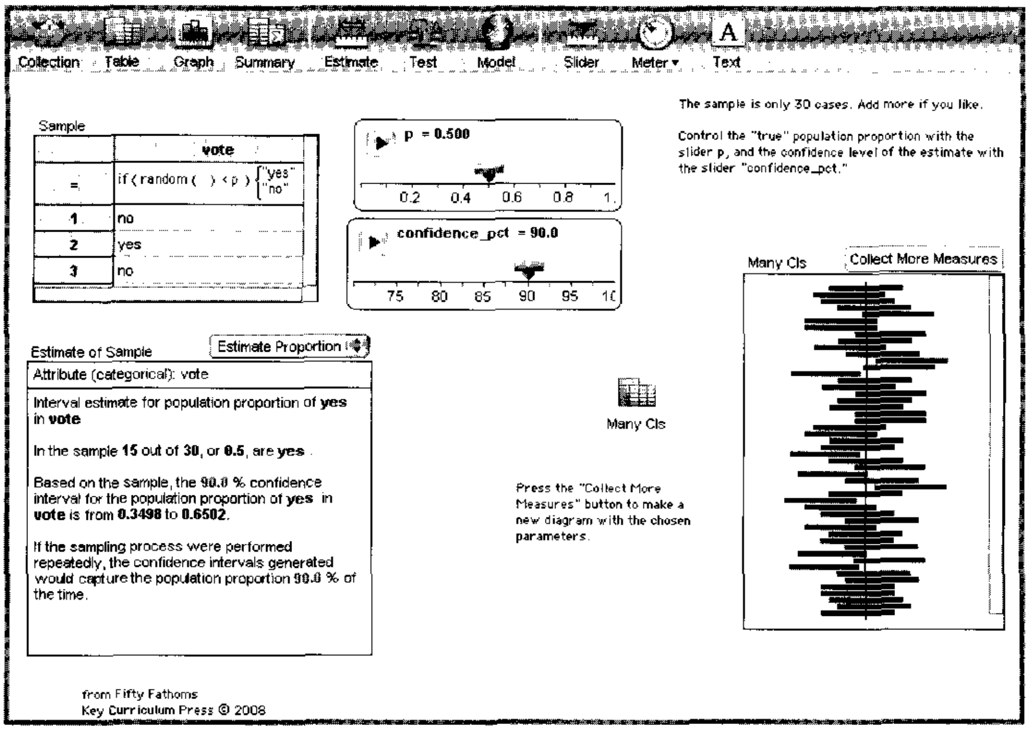
Demo 29: Capturing with Confidence Intervals

How confidence intervals of a proportion do not always capture the population value

The mantra of the 90% confidence interval is that if the process were repeated many times, 90% of the intervals we would generate would contain the [unknown] population proportion. (Many teachers are careful to point out that it does *not* mean that there's a 90% *chance* that the interval contains the proportion; other teachers—Bayesian in spirit, perhaps—wonder if there's any difference.)

In this demo, we simulate the situation, generating a lot of intervals, and see how many contain the truth. To do this, we need to know the population proportion, of course. Just remember that in real life, you get only *one* of the possible intervals, and you will have no idea which one it is.



What To Do

- ▷ Open **Capturing Props with CIs.ftm.** It should look something like the illustration.

You can see the case table showing the first three of a sample of 30 voters in the upper left. Below it, you can see an *Estimate,* a Fathom object that computes a confidence interval—in this case, a 90% confidence interval. Note the "CI mantra" in the last paragraph. At right, we see the results of pulling 50 samples from the same population and computing their confidence intervals. The true proportion—which you would never know in real life—is the vertical line. The red bars are the intervals that missed the true proportion.

Note: **Estimate of Sample** (the object with the CI in the lower left) corresponds to the *last* (bottom) bar drawn in the diagram at the lower right.

- ▷ Press the **Collect More Measures** button in the "bar" display (**Many CIs**). The display will recompute, drawing 50 new samples and computing 50 new confidence intervals.
- ▷ Repeat the process, noting (informally) how many confidence intervals miss the true proportion.
- ▷ Predict what will be different if you change from a 90% confidence interval to a 95% confidence interval.

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- ▷ To make that change, move the slider named **confidence_pct** to 95 and press **Collect More Measures** again. See if your prediction was correct. Repeat as needed.
- ▷ Do the same for an 80% confidence interval: predict, then test, setting **confidence_pct** to 80.
- ▷ Predict how it would look different if you had a 90% interval, but with a population proportion of only 0.05.
- ▷ Try that out, setting sliders **confidence_pct = 90** and **p = 0.05.**

Questions

1. About how many CIs missed the true proportion when you had the initial settings (**confidence_pct = 90** and **p = 0.5**)?
2. About how much did that number vary from experiment to experiment?
3. When you changed to a 95% confidence interval, what happened?
4. What happened with the 80% intervals?
5. What happened when you used a proportion of **p = 0.05**?

This demo may give you a different look at confidence intervals. First, it's great to see how confidence intervals can miss the true value.

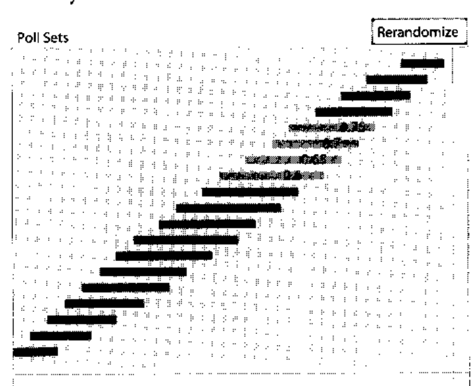
Second, it's great to see how changing the confidence level changes the look of the intervals. When you go from 90% to 80%, the intervals stay in the same places, but they get *smaller*—so the ones that *almost* miss the true proportion start to miss it. This is a confusing issue: It's easy to think, 95% is a better interval—more confidence, after all—so it should be smaller—that is, more precise. Not so.

Finally, when you change the proportion to a value near 0 (or near 1), you can see how the bars are no longer all about the same length. They are also asymmetrical when Fathom uses the exact binomial calculation, but you cannot see that on this display.

We look at how confidence intervals for the mean of a continuous attribute capture (and fail to capture) the true mean in Demo 35, "Capturing the Mean with Confidence Intervals."

Extension

Play with the document **CI Stairway 2.ftm.** This shows a famous representation of the confidence interval; follow the suggestions on the screen, whose graph appears below. Play with the three sliders to see which influence the widths of the bands; figure out how and why.



Another document, **CI Stairway.ftm,** does much the same, but it actually generates polls and so is slower but has sampling variation.

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