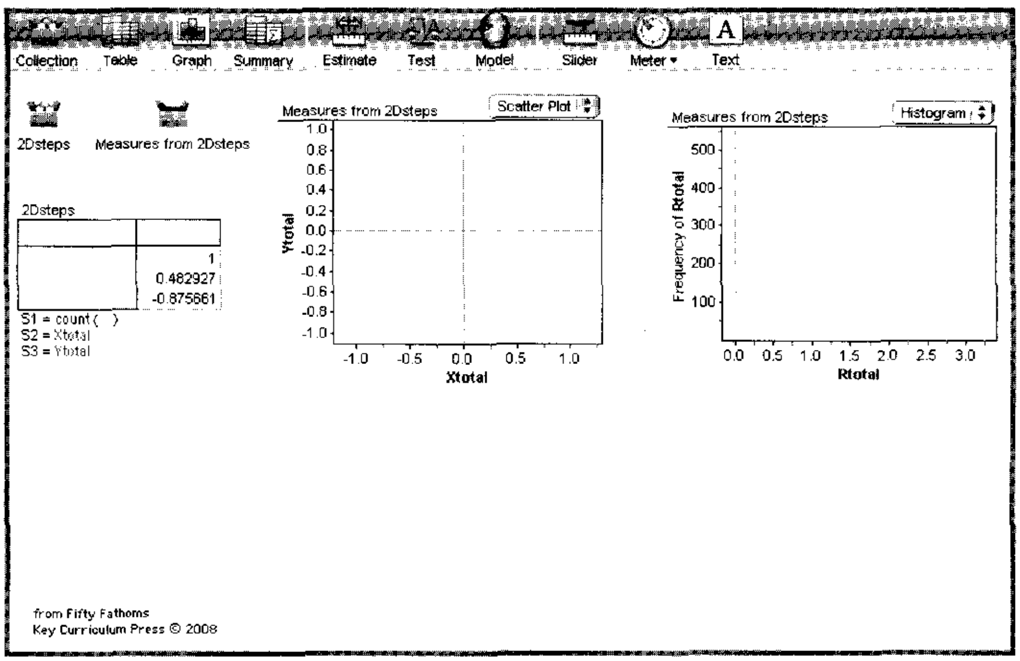
Demo 15: Two-Dimensional Random Walks

Unexpected behavior in 2D random walks • How the 2D walk eventually looks like a 2D normal distribution

This demo may be beyond the scope of most introductory statistics curricula; it is in here partly just for fun because we'll see something that is unexpected. But it also helps us think about one-dimensional random walks.

There are several ways you could imagine a two-dimensional random walk. In one way—let's call it Cartesian—you flip a coin for the horizontal axis and a coin for the vertical axis. Then you take one step along each axis, randomly positive or negative depending on the coin.

Instead, we'll do a *polar* random walk. We'll choose a direction at random, and then take one step in that direction.



What To Do

▷ Open **2D Random Walk.ftm.** It will look like the illustration.

In the upper left, you see the collection (box with gold balls) called **2Dsteps.** This contains the individual steps of our walk. The summary table below it shows the number of steps (one at first) and the ending points (**Xtotal** and **Ytotal**) of the entire walk.

The collection and graphs at right will collect and display the results from many walks. Let's make those appear.

▷ Click once on the **Measures from 2Dsteps** collection to select it.

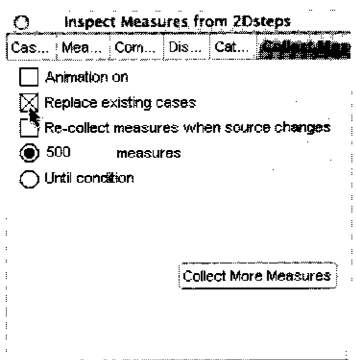
▷ Choose **Collect More Measures** from the **Collection** menu. A point appears in the scatter plot—it's the endpoint of the walk.

Note: The shortcut for **Collect More Measures** is **⌘+Y** on the Mac, **Control+Y** in Windows.

▹ Continue to collect more measures in this way, one point at a time, until you see the shape of the graph. It should be a circle—and that should make sense.

The far-right graph will probably look a little strange; it should be a spike at **Rtotal** = 1. It's a graph of the distances of the ends of the walks from the origin. In this case, all of the values should be equal to one; in practice, they won't be *exactly* equal to one due to roundoff error. You can rechoose **Histogram** from the pop-up menu in the graph to make it try to rescale itself. Just remember to select the **Measures from 2Dsteps** collection again if you want to collect more measures.

▹ Let's speed things up. Double-click that measures collection to open its inspector, and press the **Collect Measures** tab to bring up that panel.



▹ Change the settings in the panel to collect 500 measures at a time, turn off the animation, and set it to **Replace existing cases,** as shown.

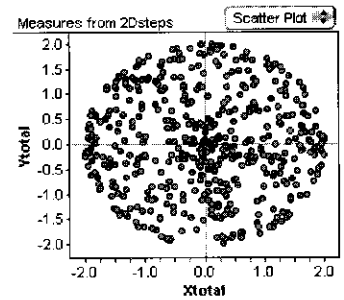
▹ Now, press the **Collect More Measures** button (it's the same as selecting and choosing the menu item, but there's enough screen space to leave the inspector open). After a bit, the graph will update to show you a nearly complete circle.

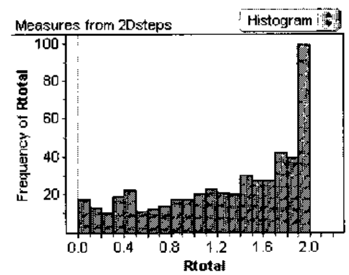
Now the real fun begins. We are going to make our walk longer: Instead of our boring one-step random walk, we'll take a *two*-step random walk!

▹ Click the original **2Dsteps** collection to select it.

▹ Choose **New Cases** from the **Collection** menu. Add one case and press **OK.** The table should update to indicate that you now have two steps in your walk.

▹ Press the **Collect More Measures** button (or select the measures collection and choose the item from the **Collection** menu). The graphs update to show you the results of 500 two-step random walks. (You may need to rechoose **Histogram** from its pop-up menu to get it to scale properly.)





Notice two things about the distribution of end points for these walks: First, there is a tight cluster near the origin; second, the histogram shows a sharp upswing as you get to a radius of 2. Think about what those walks must be like.

Note also: If there is any confusion about the correspondence between the two graphs, click on one of the bars of the histogram to select it.

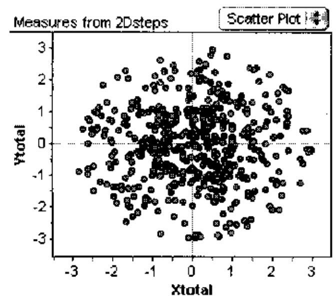
Challenges

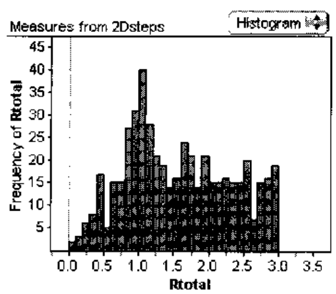
1 It should be clear that, since each step is only one unit long, all points must lie in a disk defined by $r \leq 2$, where r is the distance to the origin. What would the two graphs look like if the distribution of points were *uniform* in that disk? **Sol**

2 Use that result (or anything else you can think of) to explain why the obvious clump near the origin doesn't show up in the radial histogram, and why the "density ring" at $r = 2$ is so striking in the histogram but not in the scatter plot. **Sol**

Onward!

- Using the same procedure we used earlier, add a third case to the **2Dsteps** collection. That is, we will now have a three-step walk.
- **Collect More Measures** again. You should see graphs like the ones in the illustrations.





Notice the "density ring" at about $r = 1$, and that the whole character of the distribution has changed since we added the third step.

- Add a fourth step and collect measures. The density ring is gone!
- Add 16 steps for a total of 20. Now see what the graph looks like.

You may want to adjust the histogram's bin width. Do that by dragging the edge of a bin or by double-clicking the graph and entering new values into the inspector.

Alternatively, use an Ntigram.

- Finally, choose **Show Hidden Objects** from the **Object** menu. Now you can see a distribution where each coordinate is random normal. Note how the distribution of radii looks the same (even though the scales are different).

We haven't shown that the distribution is normal, but just as with the one-dimensional random walk—which is actually binomial—it *looks* normal after enough steps. Also, note how non-normal the distribution of *radii* is.