January 2008 SWFL Invitational Algebra I Individual Test

January 26, 2008

For each of the following questions, mark the best answer on your scantron.
If the correct answer is not present, choose E) NOTA, which denotes **None of These Answers**.

1. Solve for x: $9(2x-7)+2x\cdot 6-12\div 3+(14x-21)\div 7=2^3+9x\div 3-7\cdot 4+6^2\div 3-x(5-8)$
   A) $\frac{31}{13}$ B)$-\frac{5}{13}$ C) $\frac{40}{13}$ D) $-\frac{31}{13}$ E) NOTA

2. A quadrilateral is any closed figure with exactly four sides. The lengths of the sides of a certain quadrilateral are consecutive integers. Twelve more than twice the length of the longest side is 48 inches. Find the length of the second longest side in inches.
   A) 15 B) 16 C) 17 D) 18 E) NOTA

3. Solve for x and y:
$$3x-11y=7$$
$$-6x-14y=4$$
   A) $(\frac{9}{4},\frac{127}{12})$ B) $(\frac{1}{2},\frac{-1}{2})$ C) $(-\frac{1}{2},\frac{1}{2})$ D) $(\frac{9}{4},\frac{35}{28})$ E) NOTA

4. What is the equation of the line that passes through the points $(2,3)$ and $(-5,7)$?
   A) $7x+4y=14$ B) $4x+7y=26$ C) $4x+7y=29$ D) $4x-7y=-13$ E) NOTA

5. Simplify:
$$\left(\frac{s^5w^3}{f^{-6}l^7}\right)^{-1}\left(\frac{f^2}{w^4}\right)\left(\frac{s^2l^{-3}s^{-7}}{l^{-6}}\right)$$
   A) $s^{-10}w^{-7}f^{-4}l^{-10}$ B) $s^{-10}w^{-4}f^{-7}l^{10}$ C) $s^{-10}w^{-7}f^{-4}l^{10}$ D) $s^{10}w^7f^4l^{-10}$ E) NOTA

6. What is the distance between $(-3,27)$ and $(-5,19)$?
   A) $8\sqrt{2}$ B) $2\sqrt{17}$ C) 8 D) $2\sqrt{15}$ E) NOTA

7. What are the x- and y-intercepts of $9x - 4y = -18$?

   A) $(-2, 0)$ and $(0, \frac{9}{2})$ B) $(\frac{9}{2}, 0)$ and $(0, -2)$ C) $(-2, 0)$ and $(0, -\frac{9}{2})$

   D) $(-\frac{9}{2}, 0)$ and (0,2) E) NOTA

8. John needs to create a scale model of the Sphinx for a history project. If he decides to make his model Sphinx to the scale of 2 inches to 7 feet, how long must his model be, to the nearest hundredth if the actual Sphinx is 150 feet long?

   A) 42.85 in B) 525 in C) 0.09 in D) 42.86 in E) NOTA

9. What is the slope of a line perpendicular to $7x - 2y = -7$?

   A) $-\frac{2}{7}$ B) $\frac{2}{7}$ C) $-\frac{7}{2}$ D) $\frac{7}{2}$ E) NOTA

10. Find the midpoint of the line segment $\overline{AB}$, where A is the y-intercept of $4x - y = 7$ and B is the x-intercept of $5x + y = 10$.

    A) $(\frac{7}{8}, 5)$ B) $(1, -\frac{7}{2})$ C) $(5, \frac{7}{8})$ D) $(1, \frac{7}{2})$ E) NOTA

11. What is the equation of the line parallel to $9x - 7 = 6y$ that passes through the point $(2, -3)$?

    A) $y = \frac{2}{3}x - 4$ B) $y = \frac{2}{3}x + 4$ C) $y = \frac{3}{2}x - \frac{13}{2}$ D) $y = \frac{3}{2}x - 6$ E) NOTA

12. Solve the inequality:

    $$2(x - 7) - 5(x + 4) \geq 3(x + 1) - 1 \text{ and } 6 - \tfrac{7}{4}x \geq 3 - 2x$$

    A) $x \leq -12$ and $x \geq 6$ B) $-6 \leq x \leq \frac{4}{5}$ C) $-12 \leq x \leq -6$

    D) $-12 \leq x \leq -\frac{1}{2}$ E) NOTA

13. Sara's Scrumdidliumtious Chocolate Confections uses only 4 ingredients to make their chocolate. The ingredients are in the following proportions: 6 parts cocoa beans, 5 parts sugar, 3 parts milk, 1 part flavorings. What percent of the chocolate is milk, to the nearest tenth of a percent?

    A) 40.0% B) 33.3% C) 15.0% D) 20.0% E) NOTA

14. If $a\bigstar b$ is defined as the sum of the digits of $a^2 + b^2$, $a \divideontimes b$ is defined as $(a + 1)(b - 2)$ and $a\heartsuit b$ is defined as $b - a + 1$, then find $18\heartsuit[(4 \divideontimes 7)\bigstar(6 \divideontimes -1)]$.

    A) 0 B) 5 C) 6 D) -4 E) NOTA

15. Solve for N in the following:

    $$\left(\frac{N}{A} - G\right)(M + A) + 2 = GN - NI + 3I$$

    A)$(\frac{A}{M})(\frac{GMI+GA+2MI}{GAI-AI^2-MI-A})$ B) $(A)(\frac{-GMI-GA+2I}{GAI-I^2A-MI-A})$ C) $\frac{GMA-IMA-A-M^2}{AI}$

    D) $\frac{-GMA-GA^2+2A-3AI}{GA-IA-M-A}$ E) NOTA

16. Find $\frac{l}{s}$ if s is $\frac{6}{7}$ of w, w is $\frac{4}{3}$ of f, and f is $\frac{2}{5}$ of l.

A) $\frac{35}{16}$ B) $\frac{16}{35}$ C) $\frac{35}{9}$ D) $\frac{9}{35}$ E) NOTA

17. In which quadrant(s) would you find the solution to

$$3x - y \geq 6$$

$$x - y \leq -1$$

A) I, II, III only B) I only C) I, III, IV only D) III only E) NOTA

18. Which of the following are functions?

I. $\{(21, 9),\ (5, 7),\ (1, 2),\ (13, 11),\ (-4, 2),\ (9, 21),\ (2, 3)\}$

II. $x = y^2 + 7$

III. $y \leq x - 2$

IV. $f(x) = x^2 + 6 - 2x$

A) **I** only B) **I, IV** only C) **II, III, IV** only D) **II, III** only E) NOTA

19. What is the prime factorization of 237600?

A) $2^5 \cdot 3^3 \cdot 5^2 \cdot 11$ B) $2^4 \cdot 3^3 \cdot 5^2 \cdot 7$ C) $2^3 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11$ D) $2^3 \cdot 3^2 \cdot 5^2 \cdot 7^2 \cdot 11$ E) NOTA

20. Which of the following numbers are prime?

I. 2007

II. 251

III. 1

IV. 2008

V. 89

A) **II, V** only B) **I, III** only C) **II, III, V** only D) none are prime E) NOTA

21. $(x + 8)(x - 3)(2 - x)(11 - 9)$ is expanded to the form $Ax^3 + Bx^2 + Cx + D$. What is $\frac{|B-A|}{D-C}$?

A) -41 B) 41 C) $\frac{1}{41}$ D) $-\frac{1}{41}$ E) NOTA

22. If the solutions to $x^2 - 9 = 0$ are p and q, $p \geq q$, then find the value of

$$(q - 1)\left[\frac{p^2}{q^2} \cdot (p - 1)\right] - 2q + p - 1$$

A) 0 B) -18 C) 1 D) 4 E) NOTA

23. At Brian's birthday bash, there are 30 people present, and three desserts were offered–cake, cookies, and ice cream. Each guest could have as much of as many desserts as they desired. Three people had no dessert, six people had only cookies, six people had only cake and ice cream, x people had only cake, x people had only cake and cookies, y people had only cookies and ice cream, and y people had only ice cream. If $x > y$ and $x + y = 6$, how many people had all three desserts?

A) 4 B) 1 D) 5 C) 0 E) NOTA

24. A line passes through (-6,5) and (7,-3). Which of the following is not true?

A) the slope is $-\frac{8}{13}$

B) the x-intercept is $(\frac{17}{8}, 0)$

C) the y-intercept is $(0, \frac{17}{13})$

D)the line contains $(2, \frac{1}{13})$

E) NOTA

25. If $3x + 9 - 12(x - 1) = -3 + 6x - x(7 - 4)$, then what is the value of $6(x + 1) - 5$?

A) 5 B) 8 C) 13 D) 11 E) NOTA

26. Evaluate for $x = 2$, $y = 3$, and $z = -1$:

$$\frac{x^6y^2z^3}{x^2yz^{-2}}$$

A) 48 B) -96 C) 72 D) -48 E) NOTA

27. Lines $y = x + 1$ and $2x - y = -6$ intersect at point (a, b) in quadrant c. What is $-a + b + c$?

A) 2 B) -4 C) 4 D) -3 E) NOTA

28. If $a \bowtie b = a + ab - b - b^2$, then find the sum of the values of c if $3 \bowtie c = -12$.

A) 2 B) -2 C) 3 D) -3 E) NOTA

29. If $f(x) = 2x - 9$ and $g(x + 1) = x^2 - 3x + 6$, then what is $(-4)f(g(3))$?

A) -1 B)4 C) -4 D) 34 E) NOTA

30. What is the sum of the next three numbers in the sequence 1,1,2,4,7,11,...?

A) 49 B) 67 C) 68 D) 53 E) NOTA

SWFL Invitational - January 2008 Algebra I Team Question #1

Given $5x - 6y = 8$, find

A. the coordinate of the y-intercept

B. the slope

C. the area of the triangle formed by the line and the x- and y-axes

D. the coordinate of the x-intercept

SWFL Invitational - January 2008 Algebra I Team Question #2

Solve.

A. $17 - 6a = 2a - 25$

B. $6(b-1)^{-1} = 12(b+8)^{-1}$

C. $\frac{5c-3}{1-7c} = 4$

D. $(d-4)^2 = (d+3)^2$

SWFL Invitational - January 2008 Algebra I Team Question #3

Solve for all values of x.

A. $|x-4|=6$

B. $2x+3 \geq 4x-7$

C. $-1+5(x-1)<2(3+x)$

D. $6(7-4)=|x+1|$

SWFL Invitational - January 2008 Algebra I Team Question #4

Paula, Shawna, and Christine run a half-marathon, and all three place in the top 10. Christine finished first with a time of 104 minutes, and Paula finished before Shawna. Every runner in the top ten came in exactly two minutes after the previous finisher. If the ratio of the times of the three aforementioned girls is 13:14:15, find

A. The sum of the numerical value of their places.

B. The average of their times, in minutes.

C. If Shawna wants to finish at Paula's current race time for the next race, by what percent does Shawna need to decrease her current race time (to the nearest tenth of a percent)?

D. If Paula wants to finish at the same time as Christine for the next race (assuming Christine's race time remains constant), by what percent must Paula decrease her current race time (to the nearest tenth of a percent)?

SWFL Invitational - January 2008 Algebra I Team Question #5

What properties are shown in the following:

A. $3 + (4 + 5) = 3 + (5 + 4)$

B. $9s - [7t + (2s + 3u)] = 9s - [(7t + 2s) + 3u]$

C. $3w + w(2w + 7) = 3w + (2w^2 + 7w)$

D. $5v^{\frac{3}{4}} = 5v^{\frac{3}{4}}$

SWFL Invitational - January 2008 Algebra I Team Question #6

y is inversely proportional to x. When $x = 2$, $y = 6$.

A. Find y when $x = 4$.

B. Find x when $y = 4$.

C. Find y when $x = 1$.

D. Find x when $y = 2$.

SWFL Invitational - January 2008 Algebra I Team Question #7

Tim is in the business of making giant bags of sweet, delicious candy. His bags are a mixture of jelly beans and licorice. Each giant bag weighs 50kg. Jelly beans cost \$2/kg, and licorice costs \$3.50/kg.

A. How much would the mixture cost if 10kg of jelly beans were used?

B. If the mixture costs \$151, how many kg of licorice were used?

C. Tim made a mixture with 20kg of jelly beans. However, he accidentally switched the cost of jelly beans and licorice. What is the absolute value of the cost difference?

D. One week, the price of jelly beans doubles, and the price of licorice halves. With these new prices, how many kg of jelly beans are put into a mixture that costs \$92?

SWFL Invitational - January 2008 Algebra I Team Question #8

Determine the number of integral solutions to each of the following.

A. $|5x - 4| = 6$

B. $|2x + 7| = -1$

C. $|3x - 7| \leq 20$

D. $|4x + 5| < 21$

SWFL Invitational - January 2008 Algebra I Team Question #9

Let $a = 20$, $b = -3$, and $c = 0.4$. Evaluate, giving answers in scientific notation:

A. $\dfrac{a^6 b^{-4} a^{-1} c^3}{c^2 b^{-5} a^3}$

B. $\dfrac{2^b a c^{-4} b^0}{(\frac{1}{4}) a^{-1} c^{-5} b^{-3}}$

C. $\left(\dfrac{((c+b^2)^{\frac{1}{7}} - (-b^a)^{-4})}{\sqrt[4]{a^2 - (b^2 - c^2)^2 c^4}} \right)^0$

D. $(\sqrt[4]{a} \cdot b \cdot \sqrt{c})^4$

SWFL Invitational - January 2008 Algebra I Team Question #10

The equation of line A is $2x - 3y = -7$. The equation of line B is $-4x + 6y = -8$.

A. What is the distance between the y-intercepts of lines A and B?

B. The x-intercept of A can be written as $(x, 0)$ and the y-intercept of A can be written as $(0, y)$. What is the sum $x + y$?

C. The x-intercept of B can be written as $(x, 0)$ and the y-intercept of B can be written as $(0, y)$. What is the sum $x + y$?

D. What is the slope of B?

SWFL Invitational - January 2008 Algebra I Team Question #11

Amanda is baking large quantities of cookies. Luckily, Gary is eating large quantities of cookies. Amanda is able to bake 24 cookies in one hour, at a constant rate. Gary is able to eat 36 cookies in one hour, at a constant rate.

A. Amanda drank 7 Red Bulls, so she was awake for 24 straight hours. She decided to bake cookies the entire time! How many hours will it take Gary to eat all of the cookies that Amanda baked in that 24-hour period?

B. If Gary began eating cookies 105 minutes after Amanda started baking them, how many minutes after Gary began eating will there be no more cookies for Gary to eat?

C. Amanda's schedule for the last 24 hours was spent in the following way: $\frac{1}{3}$ of her time was spent sleeping, $\frac{3}{4}$ of her remaining time was spent in class, $\frac{3}{8}$ of this remaining time was spent watching America's Next Top Model, the rest was spent baking cookies! How many cookies did she bake in this 24 hour period?

D. Amanda was getting sleepy, so she could only bake at $\frac{1}{3}$ of her original speed. She gets her friend Scarlett to help her bake cookies! Scarlett can bake 80 cookies in one hour, but this rate decreases by 10 cookies each hour. How many cookies will this duo bake in 6 hours?

SWFL Invitational - January 2008 Algebra I Team Question #12

$a(b-c)(b+c) = b^2c^2 - ad + d(a-d)$

A. Solve for a.

B. Solve for b^2.

C. Solve for c^2.

D. Solve for d.

SWFL Invitational - January 2008 Algebra I Team Question #13

Solve.

A. $7 + 12 \div 2 - 3 \times 9$

B. $-3 - (1 + 27) + 13 \times 4$

C. $(2 + 6) \div (15 - 13)^2$

D. $8 \div 4 - 7(9 - (-31))$

SWFL Invitational - January 2008 Algebra I Team Question #14

Determine whether each of the following statements are True or False. Please write either "True" or "False." Responses of "T" or "F" will be counted as incorrect.

A. The line $3x - 2y = 6$ passes only through quadrants I, III, and IV.

B. $|2x - 4| \geq -2$ has no solutions.

C. $3 - 4 \cdot 5 + 8 \div 2^2 = -3$

D. $x^2 + 9 = (x - 3)(x + 3)$

Find the next two terms of each sequence.

A. 1, 3, 4, 7, 11, 18,...

B. 1, 0, 2, 1, 3, 2, 4,...

C. 1, -1, -5, -11, -19,...

D. 1, 4, 9, 16,...

Algebra 1 Solutions – January 2008 Invitational

1. Using the Order of Operations:
   $18x-63+12x-4+2x-3=8+3x-28+12+3x \Rightarrow 32x-70=6x-8$
   $\Rightarrow 26x=62 \Rightarrow x=\frac{31}{13}$
2. If the longest side is x, then $2x+12=48 \Rightarrow x=18$. Then the second longest side is one less, or 17 inches.
3. Multiplying the top equation by 2 gives $6x-22y=14$. Adding the top equation to the bottom gives $-22y-14y=14+4 \Rightarrow -36y=18 \Rightarrow y=\frac{-1}{2}$. Substituting this value of y back into either equation gives x = $\frac{1}{2}$. So the solution is $(\frac{1}{2},\frac{-1}{2})$
4. The slope is $\frac{7-3}{-5-2}=\frac{-4}{7}$. Using the point-slope equation we get that
   $(y-3)=\frac{-4}{7}(x-2) \Rightarrow 7y-21=-4x+8 \Rightarrow 4x+7y=29$.
5. $(\frac{s^5w^3}{f^{-6}l^7})^{-1}(\frac{f^2}{w^4})(\frac{s^2l^{-3}s^{-7}}{l^{-6}})=(\frac{f^{-6}l^7}{s^5w^3})(\frac{f^2}{w^4})(\frac{l^3}{s^5})=s^{-10}w^{-7}f^{-4}l^{10}$
6. $\sqrt{(27-19)^2+(-3-(-5))^2}=\sqrt{64+4}=\sqrt{68}=2\sqrt{17}$
7. X-int: 9x – 4(0) = -18. So (-2,0)
   Y-int: 9(0) – 4y = -18. So (0,9/2)
8. $\frac{2}{7}=\frac{x}{150} \Rightarrow x=\frac{300}{7} \approx 42.86$.
9. $7x-2y=-7 \Rightarrow y=\frac{7}{2}x+\frac{7}{2}$. So the slope of the perpendicular line is $\frac{-1}{7/2}=\frac{-2}{7}$.
10. A: 4(0) – y = 7. So A(0,-7). B: 5x + 0 = 10. So B(2,0). So the midpoint is
    $(\frac{2+0}{2},\frac{0+(-7)}{2})=(1,\frac{-7}{2})$
11. $9x-7=6y \Rightarrow y=\frac{3}{2}x-\frac{7}{6}$. So the slope of the perpendicular line is $\frac{3}{2}$. So the equation of the line is $(y-(-3))=\frac{3}{2}(x-2) \Rightarrow y+3=\frac{3}{2}x-3 \Rightarrow y=\frac{3}{2}x-6$
12. The solution to the first inequality is
    $2x-14-5x-20 \geq 3x+3-1 \Rightarrow -3x-34 \geq 3x+2 \Rightarrow -36 \geq 6x \Rightarrow x \leq -6$.
    The solution to the second inequality is $\frac{1}{4}x \geq -3 \Rightarrow x \geq -12$.
    So the overall solution is $-12 \leq x \leq -6$
13. $\frac{3}{6+5+3+1}=\frac{3}{15}=\frac{1}{5}=20\%$

14. 4x7 = (4+1)(7-2)=25. 6x(-1) = (6+1)(-1-2) = -21.
$25\text{star}(-21) = 25^2 + (-21)^2 = 1066 = 1+0+6+6 = 13$. 18star13 = 13-18+1= -4
$\frac{NM}{A} + N - GM - GA + 2 = GN - NI + 3I \Rightarrow \frac{NM}{A} + N - GN + NI = GM + GA - 2 + 3I$

15. $\Rightarrow N(\frac{M}{A} + 1 - G + I) = GM + GA + 3I - 2 \Rightarrow N = \frac{GM + GA + 3I - 2}{\frac{M}{A} + 1 - G + I} = \frac{GMA + GA^2 + 3AI - 2A}{M + A - GA + IA}$

$= \frac{-GMA - GA^2 - 3AI + 2A}{-IA + GA - M - A}$

16. $s = \frac{6}{7}w = \frac{6}{7}(\frac{4}{3}f) = \frac{6}{7}(\frac{4}{3}(\frac{2}{5}I)) = \frac{16}{35}I \Rightarrow \frac{I}{s} = \frac{35}{16}$
17. $y \le 3x - 6$ AND $y \ge x + 1$. When graphed the solution is in quadrant I only.
18. I and IV only since in each of those there is exactly one input for every output.
19. $2^5 \cdot 3^3 \cdot 5^2 \cdot 11$
20. 2007 is divisible by 3, by definition 1 is not prime, 2008 is divisible by 2. Thus only II and V.
21. $(x+8)(x-3)(2-x)(11-9) = (x^2 + 5x - 24)(-2x + 4) = -2x^3 - 6x^2 + 68x - 96$. So $\frac{|-6-(-2)|}{-96-68} = \frac{4}{-164} = \frac{-1}{41}$
22. The solutions are x = 3, -3. So p = 3, q = -3. Then
$(-3-1)(\frac{3^2}{(-3)^2}(3-1)) - 2(-3) + 3 - 1 = (-4)(2) + 6 + 2 = 0$
23. There are 7 regions. Let z be the number of people who ate all 3 desserts. Then we have that
$3 + 6 + 6 + x + x + y + y + z = 30 \Rightarrow 15 + 2(x + y) + z = 30 \Rightarrow 2(6) = 15 \Rightarrow z = 3$
24. The equation of the line is $(y-5) = \frac{-3-5}{7-(-6)}(x+6) \Rightarrow y - 5 = \frac{-8}{13}(x+6)$. If we check all of the statements, they are all true, thus E.
25. $-9x + 21 = 3x - 3 \Rightarrow 12x = 24 \Rightarrow x = 2$. So 6(2+1) – 5 = 13
26. Reducing x^4yz^5. So using the values we get $(2)^4(3)(-1)^5 = -48$
27. They intersect when $x + 1 = 2x + 6 \Rightarrow x = -5$. Substituting we get y = -5+1 = -4. The point (-5,-4) is in quadrant 3. So 5-4+3 = 4
28. The expression becomes
$3 + 3c - c - c^2 = -12 \Rightarrow c^2 - 2c - 15 = 0 \Rightarrow (c-5)(c+3) = 0 \Rightarrow c = 5, -3$. So the sum is 2.
29. $(-4)f(g(3)) = (-4)(f(4)) = (-4)(-1) = 4$
30. The sequence is +0,+1,+2,… So the next 3 numbers are 16,22,29. The sum is 67.