Diophantus of Alexandria

From the days of **Euclid** (ca 300 B.C.) to the time of **Hypatia** (A.D. 415), Alexandria, a city in northern Egypt, was the world's center of mathematical activity. Many famous mathematicians studied and lived there, among them **Diophantus** (ca A.D. 250), a heavily influential figure in the development of algebra. His most important work was *Arithmetica*, which contains impressive number theorems and about 130 diverse problems.

An interesting feature of Diophantus' algebra are his solutions for indeterminate equations. An example of an indeterminate equation is $5x + 2y = 20$, where the number of unknowns (two) exceeds the number of equations (one). Because he recognized only positive rational number solutions, some of his solutions are restricted to integers. In his search for ways to find solutions for indeterminate equations, he founded the branch of algebra we call Diophantine analysis.

Although his contributions to mathematics are of great importance, we know little about Diophantus' life other than that he lived in Alexandria—and we are reasonably certain of his age when he died. One of his admirers described Diophantus' life in an algebraic riddle, which legend tells us appeared on his tombstone.

> *Diophantus' youth lasted $\frac{1}{6}$ of his life.*
> *He grew a beard after $\frac{1}{12}$ more of his life.*
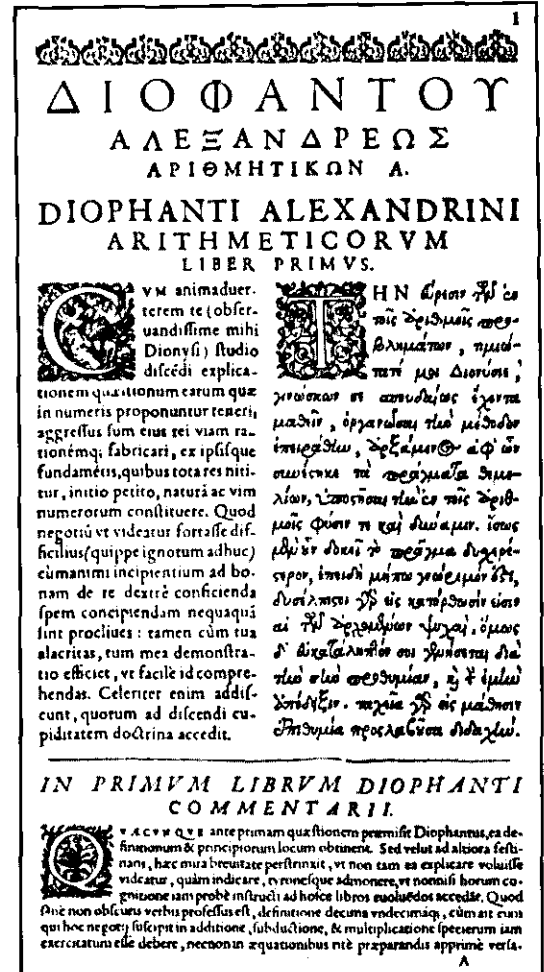> *After $\frac{1}{7}$ more of his life, Diophantus married.*
> *Five years later he had a son.*
> *The son lived exactly $\frac{1}{2}$ as long as his father.*
> *Diophantus died just four years after his son's death.*
> *All of this totals the years Diophantus lived.*

Can you solve the riddle to determine Diophantus' age at death? ★

A page from the first volume of Diophantus' six-volume Arithmetica, *written in Greek with Latin translation.*

Activities

1. Research some of the algebraic symbols introduced by Diophantus.

2. Solve these problems from *Arithmetica*.

   a. Find four numbers that, taken three at a time, add up to 22, 24, 27, and 20.

   b. In the right triangle *ABC*, right angled at *C*, *AD* bisects angle *A*. Find the set of smallest integers for *AB*, *AD*, *AC*, *BD*, and *DC* such that $DC:CA:AD = 3:4:5$.

3. When converting temperatures from Fahrenheit to Celsius, we use the formula $C = \frac{5}{9}(F - 32)$. If we substitute integer values for *F* and round the values of *C* to the nearest integer, then we are finding (approximate) Diophantine solutions. What other common formulas are used in this way?

Related Reading

Bergamini, David (ed). *Mathematics: Life Science Library*. New York: Time-Life Books, 1972.

Boyer, Carl. *A History of Mathematics*, 2nd ed rev. Uta C. Merzbach. New York: John Wiley, 1991.

Ellis, Keith. *Number Power in Nature and in Everyday Life*. New York: St. Martin's Press, 1978.

Gaughan, Edward D. "An 'Almost' Diophantine Equation." *Mathematics Teacher* (May 1980) 374–376.

Hogben, Lancelot. *Mathematics for the Millions*. New York: W.W. Norton, 1967.

Kline, Morris. *Mathematics for the Nonmathematician*. Mineola, NY: Dover, 1967.

Pappas, Theoni. *The Joy of Mathematics*. San Carlos, CA: Wide World/Tetra, 1989.

Peressini, Anthony, and Donald Sherbert. *Topics in Modern Mathematics*. New York: Holt, Rinehart and Winston, 1971.