For all questions, answer choice (E) NOTA stands for "None of These Answers"

1. $\int_0^{\frac{3}{2}} \frac{dx}{\sqrt{9-4x^2}}$

(A) $\pi/2$
(B) $\pi/4$
(C) 1/2
(D) 1
(E) NOTA

2. The inverse of f(x) = $\frac{x}{\sqrt{x^2+7}}$ can be written in the form
$f^{-1}(x) = \frac{x\sqrt{A}}{\sqrt{B-x^2}}$ with -1 < x < 1 and A and B integers.
Find the value of A + B.

(A) 10
(B) 9
(C) 8
(D) 7
(E) NOTA

3. $\lim_{x\to-\infty} \frac{68x-19}{\sqrt{6x^2+9}}$ = ?

(A) $34\sqrt{3}$
(B) -19/9
(C) $-34\sqrt{6}$
(D) $68\sqrt{6}$
(E) NOTA

4. Determine the equation of the line normal to the curve
$3(x^2 + y^2)^2 = 100xy$
at the point (3, 1).

(A) 9x + 13y = 40
(B) 2x – y = 5
(C) x + 2y = 5
(D) 3x + 2y = 11
(E) NOTA

5. If $y = Log_8 \cos^4 x$, then $\frac{dy}{dx}$ = ?

(A) $(-4\cos^3 x)(\sin x)(\ln 8)(8^{\cos^4 x})$
(B) $\frac{1}{(\ln 4096)(\cos^3 x)}$
(C) $(-4\sin x)(4\log_8 \cos^3 x)$
(D) $\frac{-4\tan x}{3\ln 2}$
(E) NOTA

6. Find the maximum area of a rectangle with perimeter M units.

(A) $M^2/16$
(B) $M^2/18$
(C) $M^2/12$
(D) $M^2/8$
(E) NOTA

7. $\lim_{x\to 0} \frac{16\sin 8x \cos 8x}{x}$ = ?

(A) 8
(B) 16
(C) 24
(D) 32
(E) NOTA

8. $\int_{\pi}^{\frac{27\pi}{2}} |\sin\theta|\, d\theta$ = ?

(A) 25
(B) 23
(C) 11.5
(D) 13
(E) NOTA

For all questions, answer choice (E) NOTA stands for "None of These Answers"

9. If f(x) = $|x^2 - 4|$, g(x) = |x|cosx and h(x) = f(x)g(x), then h'(1) = ?
   (A) 3sin1 – cos1
   (B) -3sin1 + 5cos1
   (C) -3sin1 + cos1
   (D) -5sin1 – cos1
   (E) NOTA

10. Find the area of the region between the graphs f(x) = $4(x^3 - x)$ and g(x) = 0.

   (A) -2
   (B) 0
   (C) 2
   (D) 4
   (E) NOTA

11. The displacement from equilibrium of an object in motion at time t is given by $y = \frac{1}{4}\cos(12t) - \frac{1}{3}\sin(12t)$. Determine the velocity of the object when $t = \frac{\pi}{8}$.

   (A) 3
   (B) -7
   (C) 1
   (D) -1
   (E) NOTA

12. If $y = e^{\frac{\sin(2x)}{x}}$, then y'(1) = ?

   (A) $(2e^{\sin 2})(\cos 2 - \sin 2)$
   (B) $(e^{\sin 2})(\cos 2 - 2\sin 2)$
   (C) $(e^{\sin 2})(\cos 2 + 2\sin 2)$
   (D) $(e^{\sin 2})(2\cos 2 - \sin 2)$
   (E) NOTA

13. A conical tank (with vertex down) has a diameter 14 feet across the top and 10 feet deep. If oil is flowing into the tank at a rate of 8 cubic feet per minute, find the rate of change of the depth of water (in ft/min) when the water is 6 feet deep.
   (A) $\frac{5}{24\pi}$
   (B) $\frac{200}{441\pi}$
   (C) $\frac{49}{100\pi}$
   (D) $\frac{1}{2\pi}$
   (E) NOTA

14. Find the sum of the x and y-coordinates of the point on the graph of the function f(x) = $\sqrt{x-8}$ closest to the point (2, 0).

   (A) 8
   (B) 1.5
   (C) 2
   (D) 10
   (E) NOTA

15. Use the trapezoidal rule for approximating integrals with n = 4 to *approximate* $\int_0^{\pi} (\cos^2 x)dx$.
   (A) $\frac{3\pi}{8}$
   (B) 1
   (C) $\frac{\pi}{4}$
   (D) $\frac{\pi}{2}$
   (E) NOTA

For all questions, answer choice (E) NOTA stands for "None of These Answers"

16. $\lim_{x\to 1}\dfrac{6\arctan x-\frac{3\pi}{2}}{x-1}$ = ?

(A) 3
(B) 2
(C) 1.5
(D) 1
(E) NOTA

17. Evaluate: $\lim_{x\to\infty}(1+\frac{1}{3x})^{2x}$

(A) $\frac{2}{e^3}$
(B) $\frac{e^2}{3}$
(C) $\sqrt[3]{e^2}$
(D) $\sqrt{e^3}$
(E) NOTA

18. The half-life of a substance is 398 years. If 28 grams of the substance are present in a sample initially, how much will be present after 1393 years?

(A) $\frac{7}{2}$
(B) $\frac{7\sqrt{2}}{2}$
(C) $\frac{14\sqrt{2}}{2}$
(D) $\frac{7\sqrt{2}}{4}$
(E) NOTA

19. If f(x) = $\frac{(x-2)^2}{\sqrt{x^2+1}}$ with $x \neq 2$, then $f'(0)$ = ?

(A) -4
(B) -2
(C) $-\sqrt{2}$
(D) Undefined
(E) NOTA

20. What is the minimum value of the second derivative of $y = x^4 + 6x^3 + 4x + 1$?

(A) -3/2
(B) -58
(C) -27
(D) -3
(E) NOTA

21. Which of the following is **false** about the graph of $f(x) = 2x^{5/3} - 5x^{4/3}$?

(A) Increasing and concave downward on $-\infty < x < 0$
(B) Decreasing and concave upward on $1 < x < 8$
(C) Decreasing and concave downward on $0 < x < 1$
(D) Increasing and concave downward on $8 < x < \infty$
(E) NOTA

22. $\sum_{n=1}^{16} n^2$ = ?

(A) 1512
(B) 1496
(C) 1484
(D) 1458
(E) NOTA

For all questions, answer choice (E) NOTA stands for "None of These Answers"

23. Find the average value of the function $f(x) = x^2 + 6x + 10$ over the interval [-3, -1].

(A) 7/3
(B) 3
(C) 9/2
(D) 2
(E) NOTA

24. Calculate two iterations (find x_3) of Newton's Method using $x_1 = 1$ as the initial guess to approximate a zero of $f(x) = 3x^2 - 1$.

(A) 7/12
(B) 3/5
(C) 5/9
(D) 4/7
(E) NOTA

25. If $f(\theta) = \dfrac{\sin\theta}{1-\cos\theta}$, then $f'(\theta)$ = ?

(A) $\tan\theta$
(B) $\dfrac{1}{1-\cos\theta}$
(C) $\dfrac{\cos\theta}{\cos\theta - 1}$
(D) $\dfrac{-\sin\theta}{1-\cos\theta}$
(E) NOTA

26. According to "Charlie's Law," if the temperature of a particular gas remains constant, the pressure is inversely proportional to the square of the volume. Which of the following statements is true?

(A) The rate of change of the pressure is inversely proportional to the square of the volume.
(B) The rate of change of the pressure is directly proportional to the volume.
(C) The rate of change of the pressure is inversely proportional to the cube of the volume.
(D) The rate of change of the pressure is directly proportional to the square of the volume.
(E) NOTA

27. $\int_0^{\frac{\pi}{6}} 6\sqrt{1+\tan^2 x}\,dx$ = ?

(A) 3ln(3)
(B) 4
(C) 2ln(1/3)
(D) (1/2)ln(2)
(E) NOTA

28. A population of parrots in the wild can be modeled by the function

For all questions, answer choice (E) NOTA stands for "None of These Answers"

$P(t) = \frac{100t^2}{t^2+1}$ if $t \geq 0$, where t is measured in years and P is measured in hundreds of parrots. How many years from the present time is the parrot population growing fastest?

(A) 0
(B) $\frac{1}{3}$
(C) $\frac{\sqrt{3}}{3}$
(D) 1
(E) NOTA

Who needs one of those things to do math?

29. If $y = 2x^3 - 15x^2 - 144x$, then $\frac{dy}{d(x^2-16x)} = ?$

(A) 6x + 12
(B) $6x^2 - 30x - 144$
(C) 3x + 9
(D) $x^2 - \frac{15}{2}x - 72$
(E) NOTA

30. If $\frac{A}{x+6} + \frac{B}{x-5} = \frac{x-27}{x^2+x-30}$, then B = ?

(A) 3
(B) -2
(C) -3
(D) 9
(E) NOTA

2008 Lee County Invitational **Calculus Team: Question #1**

Use differentials to *approximate* each radical. Use the function $f(x) = \sqrt{x}$ and the given values for x and dx. Write your answers as simplified improper fractions.

(A) Approximate $\sqrt{25.5}$ using x = 25 and dx = 0.5

(B) Approximate $\sqrt{49.6}$ using x = 49 and dx = 0.6

(C) Approximate $\sqrt{81.75}$ using x = 81 and dx = 0.75

(D) Approximate $\sqrt{99.4}$ using x = 100 and dx = -0.6

2008 Lee County Invitational **Calculus Team: Question #2**

Find the exact value of each limit. If the limit does not exist (or approaches positive or negative infinity) write *DNE*.

(A) $\lim_{x \to 0} \frac{x^4 + 5x - 3}{2 - \sqrt{x^2 + 4}}$

(B) $\lim_{x \to 1} \frac{x^{1/3} - 1}{x^{0.25} - x}$

(C) $\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x^3 + 8}$

(D) $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\sin x - \cos x}$

2008 Lee County Invitational **Calculus Team: Question #3**

Find each sum.

(A) $\sum_{i=1}^{18}(i^2+4)$

(B) $\sum_{i=4}^{15}(2i-3)$

(C) $\sum_{i=1}^{14}(i^3+i)$

(D) $\sum_{i=1}^{10}(i-1)^2$

2008 Lee County Invitational **Calculus Team: Question #4**

Find the *average value* of each function on the given interval.

(A) $f(x)=x-2\sqrt{x}$ on the interval [0, 4]

(B) $f(x)=x^2-4$ on the interval [0, 5]

(C) $f(x)=\frac{2}{x}$ on the interval [1, 8] [Answer *must be* in terms of ln(2)]

(D) $f(x)=\cos x-\sin x$ on the interval $[0, \frac{\pi}{6}]$

2008 Lee County Invitational **Calculus Team: Question #5**

$$h(x) = f(x)g(x) \text{ and } p(x) = \frac{f(x)}{g(x)}$$

Use the table below to find the exact values of the derivatives at the given points. Write your answers in *simplified fraction form*.

	$x = 1$	$x = 2$
$f(x)$	4	6
$g(x)$	1/3	1/2
$f'(x)$	1/4	4
$g'(x)$	-8	12
$f''(x)$	-3/2	-1
$g''(x)$	10	-2

(A) $h'(2) = ?$ (B) $p'(1) = ?$ (C) $p'(2) = ?$ (D) $h''(1) = ?$

2008 Lee County Invitational **Calculus Team: Question #6**

For each part, find the value(s) of c guaranteed by the indicated theorem. If the stated theorem does not apply, write "*does not apply*".

(A) Find all values of "c" that satisfy Rolle's Theorem for $f(x) = x^4 - 2x^2$ on the interval [-2, 2].

(B) Find all values of "c" that satisfy Rolle's Theorem for $f(x) = x - x^{\frac{1}{3}}$ on the interval [-1, 1].

(C) Find all values of "c" that satisfy the Mean Value Theorem for derivatives for $f(x) = x^2$ on the interval [-4, 1].

(D) Find all values of "c" that satisfy the Mean Value Theorem for derivatives for $f(x) = x(x^2 - 3x - 4)$ on the interval [-1, 1].

2008 Lee County Invitational **Calculus Team: Question #7**

Given the function $f(x) = x^4 - 4x^3$,

(A) On what interval(s) is $f(x)$ increasing?

(B) On what interval(s) is $f(x)$ decreasing?

(C) On what interval(s) is $f(x)$ concave upward?

(D) On what interval(s) is $f(x)$ concave downward?

2008 Lee County Invitational **Calculus Team: Question #8**

Find the exact value of each definite integral.

(A) $\int_0^2 |x-2|\,dx$

(B) $\int_0^4 |2x-3|\,dx$

(C) $\int_0^5 |8-2x|\,dx$

(D) $\int_2^6 |4-x|\,dx$

2008 Lee County Invitational **Calculus Team: Question #9**

For each of the following, find two positive numbers A and B that satisfy the given requirements. *All radicals must be in simplest form* and answers for each part should be given in the form (A, B).

(A) The product is 192 and the sum is a minimum.

(B) The product is 192 and the sum of the first and three times the second is a minimum, where A is the first number and B is the second number.

(C) The product is 108 and the sum is a minimum.

(D) The product is 108 and the sum of the first and three times the second is a minimum, where A is the first number and B is the second number.

2008 Lee County Invitational **Calculus Team: Question #10**

Given the curve: $3xy^2 + 2x^2y + 4y = xy$,

(A) Find the slope of the tangent to the curve at the point (0, 0).

(B) Find the slope of the tangent to the curve at the point (1, -5/3)

(C) Find the slope of the normal to the curve at the point (1, -5/3)

(D) Find the slope of the tangent to the curve at the point (-1, 7/3)

2008 Lee County Invitational **Calculus Team: Question #11**

For each part, find the *exact value*.

(A) If $f(x) = 12\sec x$, then $f'(\frac{7\pi}{6})$ = ?

(B) If $f(x) = -3\csc x$, then $f'(\frac{14\pi}{3})$ = ?

(C) If $f(x) = 4\sin x$, then $f'(\frac{5\pi}{12})$ = ?

(D) If $f(x) = 20\cos x$, then $f'(\frac{\pi}{12})$ = ?

2008 Lee County Invitational **Calculus Team: Question #12**

For each part, find the *exact value* of the definite integral.

(A) $\int_{1}^{5} 3^x dx$ = ?

(B) $\int_{1/2}^{5/2} \frac{x}{\sqrt{2x-1}} dx$ = ?

(C) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\cos x - 2\sec^2 x) dx$ = ?

(D) $\int_{0}^{4} 4xe^x dx$ = ?

2008 Lee County Invitational **Calculus Team: Question #13**

If $f(x) = 9x^5 + \ln(x) - \cos x$, then

(A) $f'(x)$ = ?

(B) $f''(x)$ = ?

(C) $f^{(3)}(x)$ = ?

(D) $f^{(4)}(x)$ = ?

2008 Lee County Invitational **Calculus Team: Question #14**

Given that ln(2) = 0.69 and ln(5) = 1.61, use the properties of logarithms to approximate each of the following to two decimal places.

(A) ln(20) = ?

(B) $\ln(\frac{5}{2})$ = ?

(C) $\ln(\frac{1}{40})$ = ?

(D) $\ln(\sqrt[3]{200})$ = ?

2008 Lee County Invitational **Calculus Team: Question #15**

For each of the following, find *all* points of inflection.

(A) $f(x) = x^3 - 6x^2 + 12x$

(B) $f(x) = 2x^4 - 8x + 3$

(C) $f(x) = 6x^4 - 9x^3$

(D) $f(x) = \dfrac{x+1}{\sqrt{x}}$

1. $\frac{1}{2}\int_0^{3/2}\frac{2dx}{\sqrt{9-4x^2}}=\frac{1}{2}\arcsin\frac{2(3/2)}{3}-\frac{1}{2}\arcsin\frac{2(0)}{3}=(\frac{1}{2}\arcsin 1-\frac{1}{2}\arcsin 0)=(\frac{1}{2})(\frac{\pi}{2})$ $=\frac{\pi}{4}$ => **B**

2. $y=\frac{x}{\sqrt{x^2+7}}$ => $y^2=\frac{x^2}{x^2+7}$ => $x^2y^2+7y^2=x^2$ => $\frac{7y^2}{1-y^2}=x^2$ => $\frac{y\sqrt{7}}{\sqrt{1-y^2}}=x$ … switch the x's and y's so that $f^{-1}(x)=\frac{x\sqrt{7}}{\sqrt{1-x^2}}$ => A = 7 and B = 1 => A + B = 8 => **C**

3. Multiply both top and bottom by $\frac{1}{\sqrt{x^2}}$ and the limit = $\frac{-68}{\sqrt{6}}=\frac{-34\sqrt{6}}{3}$ => **E**

4. $6(x^2+y^2)(2x+2yy')=100(xy'+y)$ …now plug in (3,1) and solve… 6(9 + 1)(6 + 2y') = 100(3y' + 1) =>360 + 120y' = 300y' + 100 =>180y' = 260 => y' = 26/18 = 13/9 => slope of normal = -9/13 and the equation of the line through (3,1) with slope -9/13 is 9x + 13y = 40 => **A**

5. $\frac{dy}{dx}=\frac{(4\cos^3 x)(-\sin x)}{(\ln 8)(\cos^4 x)}=\frac{-4\sin x}{(3\ln 2)(\cos x)}=\frac{-4\tan x}{3\ln 2}$ => **D**

6. 2x + 2y = M =>A = xy = $x(\frac{M-2x}{2})=\frac{Mx-2x^2}{2}$ => $\frac{dA}{dx}=\frac{M}{2}-2x$ …to find critical points we set equal to 0 => 0 = $\frac{M}{2}-2x$ => x = $\frac{M}{4}$ and plugging into first equation above we get y = $\frac{M}{4}$ => maximum area = $(\frac{M}{4})(\frac{M}{4})$ = $\frac{M^2}{16}$ => **A**

7. $\lim_{x\to 0}\frac{16\sin(8x)\cos(8x)}{x}=\lim_{x\to 0}\frac{16\cos(8x)}{1}\bullet\frac{8\sin(8x)}{8x}=\lim_{x\to 0}\frac{16\cos(8x)}{1}(8)(1)=\lim_{x\to 0}(16\cos(8x))(8)$ $=16(8)(\cos 0)=128\Rightarrow \text{E}$

8. $12.5\int_0^{\pi}\sin\theta d\theta=(12.5)(2)=25$ => **A**

9. h' = fg' + gf' = $(|x^2-4|)(-|x|\sin x+\frac{x}{|x|}\cos x)$ + $(|x|\cos x)(\frac{2x^3-8x}{|x^2-4|})$ … note: $\frac{d(|u|)}{dx}=\frac{u'u}{|u|}$ …plugging in x = 1 => 3(-sin1 + cos1) + (cos1)(-2) = cos1 – 3sin1 => **C**

10. Area = $\int_a^b\left(f(x)-g(x)\right)dx$, where a and b are the x coordinates of the intersection

points of the curves, and f(x) is the top curve, and g(x) is the bottom curve. There are three intersection points and for [-1,0] f(x) is the top curve, but from [0,1] g(x) is the top curve, so to find the area you evaluate

Area = $\int_{-1}^{0}\left(4x^3-4x\right)dx+\int_{0}^{1}\left(4x-4x^3\right)dx = x^4-2x^2\Big|_{-1}^{0}+2x^2-x^4\Big|_{0}^{1}=1+1=2$.=>**C**

11. y' = $-3\sin 12t-4\cos 12t$ => at t = $\frac{\pi}{8}$ we get $-3\sin\frac{3\pi}{2}-4\cos\frac{3\pi}{2}$ = -3(-1) = 3 =>**A**
12. y' = $(2x\cos 2x-\sin 2x)\,(e^{\frac{\sin 2x}{x}})$ => y'(1) = (2cos2 – sin2)($e^{\sin 2}$) => **D**
13. V = $\frac{\pi r^2 h}{3}$ and $\frac{7}{10}=\frac{r}{h}$ => r = 7h/10 => V = $\frac{49\pi h^3}{300}$ => $\frac{dv}{dt}=(\frac{49\pi}{100})(h^2)(\frac{dh}{dt})$ => 8 = $(\frac{49\pi}{100})(36)\,(\frac{dh}{dt})$ => $\frac{2}{9}=(\frac{49\pi}{100})\,(\frac{dh}{dt})$ => $\frac{dh}{dt}$ = $\frac{200}{441\pi}$ => **B**
14. Looking for critical points where f(x) is defined…on [8, ∞] we find none inside of the interval so, using the endpoint x = 8 we find that the point (8, 0) is the closest point on the graph to (2,0)…or simply draw the graph => sum = 8 => **A**
15. $\frac{\pi}{8}(\cos^2 0+2\cos^2\frac{\pi}{4}+2\cos^2\frac{\pi}{2}+2\cos^2\frac{3\pi}{4}+\cos^2\pi)=\frac{\pi}{8}(1+1+0+1+1)=\frac{\pi}{2}$ => **D**
16. L'hopital's rule => lim = $\lim_{x\to 1}\frac{\frac{6}{1+x^2}}{1}=\frac{\frac{6}{2}}{1}=3$ => **A**
17. By definition $e=\lim_{x\to\infty}(1+\frac{1}{x})^x$. This can be extended to show that $e^{a/b}=\lim_{x\to\infty}(1+\frac{1}{bx})^{ax}$. So we get $e^{2/3}$ =>**C**
18. 1393/398 = 3.5 => $28(\frac{1}{2})^{3.5}$ = $28\,(\frac{1}{8})(\frac{\sqrt{2}}{2})$ = $\frac{7\sqrt{2}}{4}$ => **D**
19. Taking the natural log of both sides we get lny =ln $\frac{(x-2)^2}{\sqrt{x^2+1}}$ =>

    lny = 2ln(x – 2) - $\frac{1}{2}\ln(x^2+1)$ => $\frac{y'}{y}=\frac{2}{x-2}-\frac{x}{x^2+1}$ =>

    y' = $\frac{(x-2)^2}{\sqrt{x^2+1}}[\frac{2}{x-2}-\frac{x}{x^2+1}]$ => y'(0) = $(\frac{4}{1})(\frac{2}{-2})$ = 4(-1) = -4 => **A**
20. To find the minimum value of the second derivative, we must set the 3rd derivative equal to zero…y' = $4x^3+18x^2+4$, y'' = $12x^2+36x$, y''' = 24x + 36 => 0 = 24x + 36 => 24x = -36 => x = -3/2 (upon testing we find this to be a minimum) => y''(-3/2) = 12(9/4) + 36(-3/2) = 27 – 54 = -27 => **C**

21. f '(x) = $\frac{10}{3}x^{1/3}(x^{1/3}-2)$ and f ''(x) = $\frac{20(x^{1/3}-1)}{9x^{2/3}}$. The function has two critical numbers at x = 0 and x = 8 and two possible points of inflection at x = 0 and x = 1. The domain is all real numbers. f '(x) is positive and f ''(x) is negative on $-\infty < x < 0$, so (A) is true. Continuing the analysis in the same way, we find (B) and (C) to be true. Since f ''(x) is positive on $8 < x < \infty$, the graph is concave upward on that interval and (D) is false => **D**

22. Sum of squares from 1 to n is given by $\frac{n(n+1)(2n+1)}{6}$ …plugging in n = 16 gives $\frac{16(17)(33)}{6}$ = (8)(17)(11) = (88)(17) = 1496 => **B**

23. $\frac{1}{-1-(-3)}\int_{-3}^{-1}(x^2+6x+10)dx = \frac{1}{2}[\frac{x^3}{3}+3x^2+10x]$ evaluated from -3 to -1 =
$\frac{1}{2}[(\frac{-1}{3}+3-10)-(-9+27-30)] = \frac{1}{2}(\frac{-22}{3}+12) = \frac{1}{2}(\frac{14}{3}) = \frac{7}{3}$ => **A**

24. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ and f '(x) = 6x. $x_2 = 1-\frac{2}{6} = \frac{2}{3}$ =>
$x_3 = \frac{2}{3}-\frac{1/3}{4} = \frac{2}{3}-\frac{1}{12} = \frac{7}{12}$ => **A**

25. $\frac{(1-\cos\theta)(\cos\theta)-(\sin\theta)(\sin\theta)}{(1-\cos\theta)^2} = \frac{\cos\theta-\cos^2\theta-\sin^2\theta}{(1-\cos\theta)^2} = \frac{\cos\theta-1}{(1-\cos\theta)^2} = \frac{-1}{1-\cos\theta}$
=> **E**

26. $P = \frac{Tk}{V^2}$ => $P' = \frac{-2Tk}{V^3}$ => **C**

27. $= 6\int_0^{\frac{\pi}{6}} \sec x dx$ (note: we can eliminate the absolute value when removing from the radical because it's positive from 0 to $\pi/6$) = $6\ln|\sec x + \tan x|$ evaluated from 0 to $\pi/6$ = $6(\ln|\frac{2\sqrt{3}}{3}+\frac{\sqrt{3}}{3}|-\ln|1|)$ = $6\ln\sqrt{3} = 3\ln 3$ => **A**

28. The parrot population is growing fastest when the 1st derivative is at a maximum. So $P'(t)=\frac{200t(t^2+1)-100t^2(2t)}{(t^2+1)^2}=\frac{200t}{(t^2+1)^2}$, then $P''(t)=\frac{200(t^2+1)^2-200t[2(t^2+1)(2t)]}{(t^2+1)^4}=\frac{200(t^2+1)[(t^2+1)-2(2t)(t)]}{(t^2+1)^4}=\frac{200(-3t^2+1)}{(t^2+1)^3}$. P''(t) has a critical value when the numerator is 0 (the denominator cannot be 0 since setting it equal to 0 gives imaginary roots). So the critical values we get by setting the numerator equal to 0 are $t=\pm\sqrt{\frac{1}{3}}$. By the domain restrictions, the only possible t is $t=\sqrt{\frac{1}{3}}=\frac{\sqrt{3}}{3}$. Testing this value shows that this is in fact the value that maximizes P''. => **C**

29. $\frac{dy}{d(x^2-16x)}=\frac{dy/dx}{d(x^2-16x)/dx}=\frac{6x^2-30x-144}{2x-16}=$ $\frac{3x^2-15x-72}{x-8}=\frac{(x-8)(3x+9)}{x-8}=3x+9$ => **C**

30. $A(x-5)+B(x+6)=x-27$. Plugging in x = 5 to eliminate A, we get 11B = -22 => B = -2 => **B**

1. B
2. C
3. E
4. A

5. D
6. A
7. E
8. A
9. C
10. C
11. A
12. D
13. B
14. A
15. D
16. A
17. C
18. D
19. A
20. C
21. D
22. B
23. A
24. A
25. E
26. C
27. A
28. C
29. C
30. B

1. f(x) + f '(x)dx = $\sqrt{x}+\frac{1}{2\sqrt{x}}$ dx

(A) $5+(\frac{1}{10})(\frac{1}{2})=\boxed{\frac{101}{20}}$

(B) $7+(\frac{1}{14})(\frac{3}{5})=\boxed{\frac{493}{70}}$

(C) $9+(\frac{1}{18})(\frac{3}{4})=9+\frac{1}{24}=\boxed{\frac{217}{24}}$

(D) $10+(\frac{1}{20})(\frac{-3}{5})=10-\frac{3}{100}=\boxed{\frac{997}{100}}$

2. (A) -3/0 => Indeterminate => $\boxed{\text{DNE}}$

(B) L'hopital's Rule => $\frac{\frac{1}{3}x^{-2/3}}{\frac{1}{4}x^{-3/4}-1}=\frac{1/3}{-3/4}=\boxed{\frac{-4}{9}}$

(C) L'hopital's Rule (or factoring) => $\frac{\frac{-1}{x^2}}{3x^2}=\frac{-1/4}{12}=\boxed{\frac{-1}{48}}$

(D) L'hopital's Rule => $\frac{-\sec^2 x}{\cos x+\sin x}=\frac{-2}{\sqrt{2}}=\boxed{-\sqrt{2}}$

3. (A) $\frac{n(n+1)(2n+1)}{6}+(18)(4)=\frac{3(19)(37)}{6}+(18)(4)=2109+72=\boxed{2181}$

(B) $\frac{12}{2}(5+27)=(6)(32)=\boxed{192}$

(C) $[(15)(7)]^2+(15)(7)=105^2+105=\boxed{11130}$

(D) $i^2-2i+1=\frac{(10)(11)(21)}{6}-(2)(5)(11)+10=(35)(11)-110+10=\boxed{285}$

4. (A) $\frac{1}{4}\int_0^4(x-2\sqrt{x})dx=\frac{1}{4}(8-\frac{32}{3})=(\frac{1}{4})(\frac{-8}{3})=\boxed{\frac{-2}{3}}$

(B) $\frac{1}{5}\int_0^5(x^2-4)dx=\frac{1}{5}(\frac{125}{3}-20)=(\frac{25}{3}-4)=\frac{13}{3}$

(C) $\frac{1}{7}\int_1^8\frac{2}{x}dx=\frac{2}{7}(\ln 8-\ln 1)=\frac{2}{7}\ln 8=\boxed{\frac{6}{7}\ln 2}$

(D) $\frac{1}{\frac{\pi}{6}-0}\int_0^{\frac{\pi}{6}}(\cos x-\sin x)dx=\frac{6}{\pi}[\frac{1}{2}+\frac{\sqrt{3}}{2}-1]=\frac{3\left(\sqrt{3}-1\right)}{\pi}$

5. h'(x) = f '(x)g(x) + f(x)g'(x), h ''(x) = f '(x)g'(x) + g(x)f ''(x) + f(x)g''(x) + g'(x)f '(x),
p'(x) = $\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
(A) f '(2)g(2) + f(2)g'(2) = (4)(1/2) + (6)(12) = $\boxed{74}$
(B) $\frac{g(1)f'(1)-f(1)g'(1)}{[g(1)]^2} = \frac{(1/3)(1/4)-(4)(-8)}{(1/3)^2} = \frac{\frac{1}{12}+32}{1/9} = \frac{(385)(9)}{12} = \boxed{\frac{1155}{4}}$
(C) $\frac{g(2)f'(2)-f(2)g'(2)}{[g(2)]^2} = \frac{(1/2)(4)-(6)(12)}{1/4} = (-70)(4) = \boxed{-280}$
(D) f '(1)g'(1) + g(1)f "(1) + f(1)g"(1) + g'(1)f '(1) = 2(1/4)(-8) + (1/3)(-3/2) + (4)(10)
= -4 – 1/2 + 40 = 36 – 1/2 = $\boxed{\frac{71}{2}}$

6. (A) $4x^3-4x=0 \Rightarrow 4x(x^2-1)=0$ => x = c = $\boxed{-1, 0, 1}$
(B) f '(x) = $1-\frac{1}{3}x^{-2/3}$ => not differentiable at x = 0 => $\boxed{\text{Does Not Apply}}$
(C) f '(c) = 2c = $\frac{1-16}{1-(-4)} = \frac{-15}{5} = -3$ => c = $\boxed{\frac{-3}{2}}$
(D) f '(c) = $3c^2-6c-4 = \frac{-6-0}{2} = -3$ => $3c^2-6c-1=0$ => c = $\frac{3+2\sqrt{3}}{3}$ and $\frac{3-2\sqrt{3}}{3}$
by quadratic formula…but only $\boxed{\frac{3-2\sqrt{3}}{3}}$ is in the interval [-1, 1]

7. f '(x) = $4x^3-12x^2 = 4x^2(x-3)$ and f ''(x) = $12x^2-24x = 12x(x-2)$
critical points at x = 0 and x = 3 and possible points of inflection at x = 0 and x = 2

(A) f ' is positive on $(3,\infty)$ so increasing on $\boxed{(3,\infty)}$
(B) f ' is negative on $(-\infty,3)$ so decreasing on $\boxed{(-\infty,3)}$
(C) f '' is positive on $(-\infty,0)$ and $(2,\infty)$ so concave up on $\boxed{(-\infty,0)\cup(2,\infty)}$
(D) f '' is negative on (0, 2) so concave down on $\boxed{(0, 2)}$

8. (A) = $\int_0^2 (2-x)dx$ = 4 – 4/2 = $\boxed{2}$
(B) = $\int_0^{3/2} (3-2x)dx + \int_{3/2}^{4} (2x-3)dx$ = 9/2 – 9/4 + 16 – 12 – (9/4 – 9/2) = 9 + 4 – 9/2 = $\boxed{\frac{17}{2}}$
(C) = $\int_0^4 (8-2x)dx + \int_4^5 (2x-8)dx$ = 32 – 16 + 25 – 40 – (16 – 32) = $\boxed{17}$
(D) = $\int_2^4 (4-x)dx + \int_4^6 (x-4)dx$ = 16 – 8 – (8 – 2) + 18 – 24 – (8 – 16) = $\boxed{4}$

9. (A) xy = 192 and x + y = S => y = 192/x => x + 192/x = S => 1 - $\frac{192}{x^2}$ = S' = 0
=> x = $\sqrt{192}$ (looking for positive value) => y = $\sqrt{192}$
(question says that radicals must be simplified) => (A, B) = $\boxed{(8\sqrt{3}, 8\sqrt{3})}$

(B) xy = 192 and x + 3y = S…using the same substitution procedure as above…
(A, B) = $\boxed{(24, 8)}$
(C) Same substitution procedure as in part (A) above => (A, B) = $\boxed{(6\sqrt{3}, 6\sqrt{3})}$
(D) Same substitution procedure as in part (B) above => (A, B) = $\boxed{(18, 6)}$

10. $(3x)(2yy') + (3y^2) + 2x^2y' + 4xy + 4y' = xy' + y$ => $6xyy' + 2x^2y' + 4y' - xy' = y - 4xy - 3y^2$
=> $y'(6xy + 2x^2 + 4 - x) = y - 4xy - 3y^2$ => $y' = \frac{y - 4xy - 3y^2}{6xy + 2x^2 + 4 - x}$
(note: all points listed are on the curve)
(A) plugging in (0, 0) into y', we get y' = 0/4 = $\boxed{0}$
(B) plugging in (1, -5/3) into y', we get y' = $\frac{\frac{-5}{3} + \frac{20}{3} - \frac{25}{3}}{-10 + 2 + 4 - 1} = \frac{\frac{-10}{3}}{-5} = \boxed{\frac{2}{3}}$
(C) using part (B), we take the negative reciprocal and get $\boxed{\frac{-3}{2}}$
(D) plugging in (-1, 7/3) into y', we get y' = $\frac{\frac{7}{3} + \frac{28}{3} - \frac{49}{3}}{-14 + 2 + 4 + 1} = \frac{\frac{-14}{3}}{-7} = \frac{2}{3}$

11. (A) f '($\frac{7\pi}{6}$) = 12(sec$\frac{7\pi}{6}$)(tan$\frac{7\pi}{6}$) = (12) ($\frac{-2}{\sqrt{3}}$)($\frac{\sqrt{3}}{3}$) = $\boxed{-8}$
(B) f '($\frac{14\pi}{3}$) = -3(-csc$\frac{14\pi}{3}$)(cot$\frac{14\pi}{3}$) = (3)($\frac{2}{\sqrt{3}}$)($\frac{-1}{\sqrt{3}}$) = $\boxed{-2}$
(C) f '($\frac{\pi}{4} + \frac{\pi}{6}$) = 4cos($\frac{\pi}{4} + \frac{\pi}{6}$) = 4($\frac{\sqrt{6} - \sqrt{2}}{4}$) = $\boxed{\sqrt{6} - \sqrt{2}}$ (using sum/difference)
(D) f '($\frac{\pi}{4}$ - $\frac{\pi}{6}$) = -20cos($\frac{\pi}{4}$ - $\frac{\pi}{6}$) = -20($\frac{\sqrt{6} - \sqrt{2}}{4}$) = $\boxed{-5(\sqrt{6} - \sqrt{2}) \textbf{ or } 5(\sqrt{2} - \sqrt{6})}$

12. (A) = $\frac{(3^5 - 3^1)}{\ln(3)} = \boxed{\frac{240}{\ln(3)}}$
(B) letting u = 2x – 1, we get x = $\frac{u+1}{2}$ and dx = $\frac{du}{2}$ …changing our integral…
$\frac{1}{4}\int_0^4 \frac{u+1}{u^{1/2}}du = \frac{1}{4}\int_0^4 (u^{1/2} + u^{-1/2})du = \frac{1}{4}(\frac{16}{3} + 4) = \boxed{\frac{7}{3}}$

(C) = $(\sin\frac{\pi}{3} - 2\tan\frac{\pi}{3}) - (\sin\frac{\pi}{4} - 2\tan\frac{\pi}{4}) = \frac{\sqrt{3}}{2} - 2\sqrt{3} - \frac{\sqrt{2}}{2} + 2 = \boxed{\frac{4-\sqrt{2}-3\sqrt{3}}{2}}$

(D) using integration by parts with u = x and dv = $e^x dx$, the integral is equivalent to $4[xe^x - \int e^x dx] = 4e^x(x-1)$ evaluated from 0 to 4 = $\boxed{12e^4 + 4 \textbf{ or } 4(3e^4+1)}$

13. (A) $f'(x) = \boxed{45x^4 + \frac{1}{x} + \sin x}$

(B) $f''(x) = \boxed{180x^3 + \frac{-1}{x^2} + \cos x}$

(C) $f^{(3)}(x) = \boxed{540x^2 + \frac{2}{x^3} - \sin x}$

(D) $f^{(4)}(x) = \boxed{1080x + \frac{-6}{x^4} - \cos x}$

14. (A) ln(20) = ln(4) + ln(5) = 2ln(2) + ln(5) = 2(0.69) + 1.61 = $\boxed{2.99}$

(B) $\ln(\frac{5}{2})$ = ln(5) – ln(2) = 1.61 – 0.69 = $\boxed{0.92}$

(C) $\ln(\frac{1}{40})$ = ln(1) – ln(40) = 0 – (ln(2^3) + ln(5) = - [3ln(2) + ln(5)] = -[(3)(0.69) + 1.61] = $\boxed{-3.68}$

(D) = (1/3)ln(200) = (1/3)[3ln(2) + 2ln(5)] = (1/3)[(3)(0.69) + (2)(1.61)] = 5.29/3 = $\boxed{1.76}$ (to two decimal places as stated in the question)

15. (A) $\boxed{f''(x) = 6x - 12}$ => x = 2 is a possible point of inflection…checking for change of sign confirms that the point (2,8) is the only point of inflection.

(B) $f''(x) = 24x^2$ => x = 0 is a possible point of inflection…checking for change of sign, we determine that there is $\boxed{\text{no point of inflection}}$

(C) $f''(x) = 72x^2 - 54x = 18x(4x-3)$ => x = 0 and x =3/4 are possible points of inflection …checking for change of sign confirms that $\boxed{\text{(0, 0) and (3/4, -243/128)}}$ are both points of inflection.

(D) $f''(x) = \frac{-1}{4}x^{-3/2} + \frac{3}{4}x^{-5/2} = \frac{-1}{4}x^{-3/2}(1 - \frac{3}{x})$ => x = 0 and x = 3 are possible points of inflection…checking for change of sign we find that only $\boxed{(3, \frac{4\sqrt{3}}{3})}$ is a point of inflection.

1. (A) $\frac{101}{20}$ (B) $\frac{493}{70}$ (C) $\frac{217}{24}$ (D) $\frac{997}{100}$

2. (A) DNE (B) $\frac{-4}{9}$ (C) $\frac{-1}{48}$ (D) $-\sqrt{2}$

3. (A) 2181 (B) 192 (C) 11130 (D) 285

4. (A) $\frac{-2}{3}$ (B) $\frac{13}{3}$ (C) $\frac{6}{7}\ln 2$ (D) $\frac{3\left(\sqrt{3}-1\right)}{\pi}$

5. (A) 74 (B) $\frac{1155}{4}$ (C) -280 (D) $\frac{71}{2}$

6. (A) -1, 0, 1 (in any order) (B) Does Not Apply (C) $\frac{-3}{2}$ (D) $\frac{3-2\sqrt{3}}{3}$

7. (A) $(3,\infty)$ or $[3,\infty)$ (B) $(-\infty,3)$ or $(-\infty,3]$ (C) $(-\infty, 0)\cup(2, \infty)$ (D) (0, 2)

8. (A) 2 (B) $\frac{17}{2}$ (C) 17 (D) 4

9. (A) $(8\sqrt{3},\ 8\sqrt{3})$ (B) (24, 8) (C) $(6\sqrt{3},\ 6\sqrt{3})$ (D) (18, 6)

10. (A) 0 (B) $\frac{2}{3}$ (C) $\frac{-3}{2}$ (D) $\frac{1}{3}$

11. (A) – 8 (B) – 2 (C) $\sqrt{6}-\sqrt{2}$ or $2\sqrt{2-\sqrt{3}}$
    (D) $5(\sqrt{2}-\sqrt{6})$ or $-10\sqrt{2-\sqrt{3}}$ {*or an equivalent form*}

12. (A) $\frac{240}{\ln(3)}$ (C) $\frac{4-\sqrt{2}-3\sqrt{3}}{2}$ {*or an equivalent form*}
    (B) $\frac{7}{3}$ (D) $12e^4+4$ {*or* $4(3e^4+1)$ }

13. (A) $45x^4+\frac{1}{x}+\sin x$ (C) $540x^2+\frac{2}{x^3}-\sin x$
    (B) $180x^3+\frac{-1}{x^2}+\cos x$ (D) $1080x+\frac{-6}{x^4}-\cos x$

14. (A) 2.99 (B) 0.92 (C) – 3.68 (D) 1.76

15. (A) (2,8) (B) No Point of Inflection (C) (0, 0) and (3/4, -243/128) (D) $(3, \frac{4\sqrt{3}}{3})$

Individual

1. B
2. C
3. E
4. A
5. D
6. A
7. E
8. A
9. C
10. C
11. A
12. D
13. B
14. A
15. D
16. A
17. C
18. D
19. A
20. C
21. D
22. B
23. A
24. A
25. E
26. C
27. A
28. C
29. C
30. B

Team

1.
    a.