

Geometry Individual

Answers:

1) D

The area of the large square is 64. By rotating the inner square 45 degrees and drawing the diagonals of the inner square, the area of the inner square is seen to be half of the area of the outer square.

2) C

Draw a segment from the center of the circle to a vertex of the triangle and a segment from the center of the circle to where the side of the triangle is tangent to the circle so as to form a 30-60-90 right triangle. The side opposite the 60 degree angle is $\frac{1}{2}$ the length of a side of the triangle: 4. The length of the side opposite the 30 degree angle (which is the radius of the circle) is therefore, $\frac{4\sqrt{3}}{3}$.

3) D

Draw the altitude from the unmarked angle, forming a 30-60-90 right triangle and a 45-45-90 right triangle. The sides opposite the two 45 degree angles are $4\sqrt{2}$. Since the side opposite the 30 degree angle is $4\sqrt{2}$, the side opposite the unmarked 60 degree angle is $4\sqrt{6}$. Therefore, $x = 4\sqrt{2} + 4\sqrt{6}$

4) C

5) E

I, II and III are not true in the case of kite shapes that may be drawn with perpendicular and congruent diagonals.

6) A

III is not true if the points are collinear.

7) E

The cone will have the volume $V_{Co} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 8$ and the cylinder will have the

volume $V_{Cy} = \pi r^2 h$. If the volumes are equal, then $\frac{1}{3}\pi r^2 8 = \pi r^2 h$. Solving for h yields

that $h = \frac{8}{3}$.

8) D

$3x = 6 \cdot 8$. Solve: $x = 16$

9) C

Using the Pythagorean Theorem, you can find that the length of the diagonals of the rectangle is 10. Since the rectangle is inscribed in a circle, each vertex of the rectangle, which is a right angle, intercepts a semicircle. Therefore, the diagonal of the rectangle is a diameter of the circle. The radius of the circle is therefore 5, and the area is therefore 25π .

10) B

11) B

The area of the circle is 64π . By drawing segments from its center, the hexagon can be divided into 12 30-60-90 right triangles where the hypotenuse is the radius of the circle, which is 8. The sides opposite the 30 degree angles are 4 and the sides opposite the 60 degree angles are $4\sqrt{3}$. Each right triangle will have an area of $8\sqrt{3}$, so the hexagon will

have a total area of $96\sqrt{3}$. The total area of the white sections will be the area of the circle minus the area of the hexagon, or $64\pi - 96\sqrt{3}$. So $a + b + c = 163$.

12) B

$$A+B = 90. A+C = 180. B+C = 180.$$

$$A+C - (A+B) = 180 - 90.$$

$$C-B = 90.$$

$$B+C + (C-B) = 180 + 90.$$

$$2C = 270$$

$$C = 135.$$

$$A + 135 = 180.$$

$$A = 45$$

13) E

The sum of an exterior angle and an interior angle is 180. Therefore, if the exterior angle is an integer, the interior angle must also be an integer. Therefore, there are no regular polygons with exterior angles that are integers and interior angles that are not integers.

14) A

The area of the small pizza is 25π . The area of the medium pizza is 36π . The area of the large pizza is 49π . The pizza per dollar for the small pizza is $\frac{25\pi}{7}$, which is

approximately 3.5π . The pizza per dollar for the medium pizza is $\frac{36\pi}{11}$, which is

approximately 3.2π . The pizza per dollar for the large pizza is $\frac{49\pi}{11}$, which is

approximately 2.8π .

15) D

$$AE \cdot BE = CE \cdot DE$$

$$(x+3)(x-4) = (x-3)(x+1)$$

$$x = 9$$

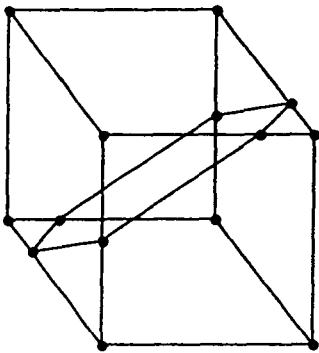
$$CE = 9-3 = 6$$

$$DE = 9+1 = 10$$

$$CD = 16$$

16) A cross-section is formed by a plane intersecting the cube. The intersection of the plane with a face of a cube is a segment. By intersecting each of the six faces of the cube with a plane, a hexagon can be formed, as seen in the diagram.

C



17) E

I, II and III are not true if C is between A and B.

18) A

$$(AX)^2 = (BX)(XC)$$

19) D

If you inscribe a regular polygon with one billion sides in a circle, the apothem of the polygon is extremely close to the radius of the circle, and the area of the polygon is extremely close to the area of the circle. Therefore, the area of the regular polygon with an apothem of length 3 can be approximated (and certainly when you are only rounding to the nearest unit) by the area of the circle with a radius of length 3.

$$\pi r^2 = 3.1(3)^2 = 27.9 \approx 28$$

20) C

The longest diagonal of a hexagonal base with edges of length 8 is 16, which can be seen when the base is divided into six equilateral triangles. By forming a right triangle with the aforementioned diagonal of length 16 and a lateral edge with length 8, the hypotenuse would be the longest diagonal of the prism, and its length, $8\sqrt{5}$, can be found with the Pythagorean Theorem.

21) D

IV is only true if ABCD is a rectangle.

22) A

The two semicircular ends can be combined to form a complete circle with circumference $\pi d = 100\pi$. The length of the two straight-aways is x . $2x + 100\pi = 1320$ can be solved to find $x = 660 - 50\pi$

23) C

24) B

By drawing a line parallel to both j and k through the vertex with measure x , the unknown angle is split into an angle that is an alternate interior angle with the 20 degree angle and an angle that is a consecutive (same-side) interior angle with the 110 degree angle. The measures of those two angles are 20 and 70 respectively, so the $x = 20 + 70 = 90$.

25) C

One method: Let line k be perpendicular to $y = 2x + 5$ and intersect $y = 2x + 5$ at its y -intercept. Then, $k: y = -\frac{1}{2}x + 5$. Next, find where this line intersects $y = 2x + 10$.

Solve $-\frac{1}{2}x + 5 = 2x + 10$, and find that $x = -2$. Substituting x back into $y = 2x + 10$, you find that the point of intersection is $(-2, 6)$. The distance between the two lines is the distance between the y -intercept $(0, 5)$ and $(-2, 6)$. Using the Distance Formula or Pythagorean Theorem, you can find that this distance is $\sqrt{5}$.

26) D

The area covered by the dog can be broken down into $\frac{3}{4}$ of a circle with radius 30 and two quarter-circles with radius 10.

27) B

28) C

I. Construct segment JK with length AB. Construct a line perpendicular to JK through J. Construct a circle with radius CD and center K. Label a point of intersection of the circle and the perpendicular line L. Triangle JKL.

II. Construct segment JK with length AB on a line. Construct a KL with length CD such that K is between J and L. Construct a circle with center J and length JL.

III. Construct segment JK with length AB. Construct lines perpendicular to JK through J and K. Construct segments JL and KM with length CD on the perpendicular lines.

Rectangle JKML has area $(AB)(CD)$

IV. This is the classic construction “circling the square”, which has been proven impossible.

29) C

The surface area of a sphere is $4\pi r^2$. The surface area of a hemisphere is $3\pi r^2$, and since you now have two of them, the new surface area is $6\pi r^2$, which is an increase of 50%.

30) A

Using Pythagorean Theorem, you can find that the hypotenuse of triangle A is $\sqrt{2}$, the hypotenuse of triangle B is $\sqrt{3}$, the hypotenuse of triangle C is $\sqrt{4}$, and so on. The hypotenuse of triangle D is $\sqrt{27} = 3\sqrt{3}$.