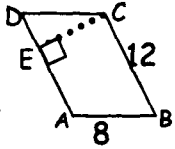


Geometry Individual Test

March Regional Competition

The abbreviation NOTA denotes "None of These Answers." Diagrams may not be drawn to scale.

1.



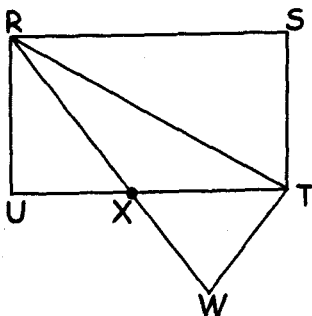
Parallelogram ABCD has perpendicular \overline{CE} drawn from point C to \overline{DA} . If $CE=6$ then the area of parallelogram ABCD is ___.

- A. 96 B. 72
C. 48 D. 36 E. NOTA

2. A right triangle has legs 6 cm and 8 cm. If its area is A sq. cm and its perimeter is B cm then find the value of A-B.

- A. 0 B. 2
C. 4 D. $\sqrt{3}-1$ E. NOTA

3.



Consider rectangle RSTU and right $\triangle RTW$ with hypotenuse \overline{RW} . If

$m\angle STR = 50^\circ$ and $m\angle RXU = 70^\circ$ then $m\angle TRX - m\angle URX =$

- A. 5 B. 10
C. 15 D. 20 E. NOTA

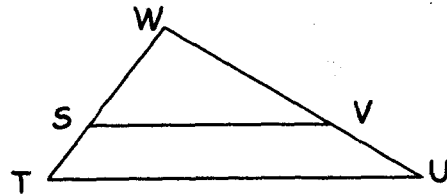
4. \overline{AB} bisects $\angle RAN$. If $m\angle BAR = 3x^\circ$ and $m\angle RAN = (8x-12)^\circ$ then find the value of $x+20$.

- A. 6 B. 12
C. 22 D. 26
E. NOTA

5. Isosceles triangle RST has base \overline{RS} . If $m\angle T = x+20$ then which is an expression for $m\angle S$?

- A. $160-2x$ B. $100-0.5x$
C. $80-0.5x$ D. $80-x$
E. NOTA

6.



In $\triangle WTU$, \overline{SV} is drawn parallel to \overline{TU} as shown. $SV=10$, $ST=3$, $TU=14$. Find length SW.

- A. 7.5 B. 7.0
C. 6.2 D. 4.2
E. NOTA

7. Chord \overline{AB} is a distance of 14 units from the center of circle R. If the radius of R is 16 units, then tell the length of \overline{AB} to the nearest tenth place.

- A. 15.5 B. 21.3
C. 30.0 D. 60.0
E. NOTA

Geometry Individual Test

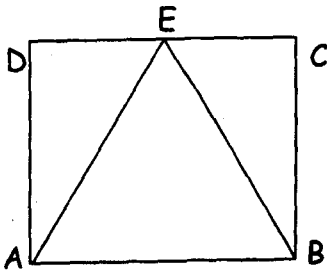
March Regional Competition

8. In rectangle $RSTU$, \overline{RT} and \overline{SU} intersect at point E . If $RE = 3x - 10$, $ES = x + 6$ and $TE = 2x - 2$, then find the length RT .
- A. 32 B. 28
C. 16 D. 12
E. NOTA
9. In triangle ABC , C is a right angle, and $m\angle A = 30^\circ$. If the perimeter of ABC is $24\sqrt{3} + 72$ then find the area of ABC .
- A. 144
B. $144\sqrt{3}$
C. $288\sqrt{3}$
D. $576\sqrt{3}$
E. NOTA
10. The longest length chord of circle C has length 20. Which must be true?
- A. The shortest chord of circle C has length 10.
B. The longest chord is distance 1 from the center C .
C. Circle C has radius 10.
D. The longest chord has length 5 more than the radius of C .
E. NOTA
11. A regular polygon has one interior angle which measures 108° more than one exterior angle. If the polygon has N sides then what is the value of $N^2 - N$?
- A. 5 B. 20
C. 70 D. 90
E. NOTA
12. Consider the set S of all parallelograms, and the set T of all rhombuses, and the set U of all rectangles. Which is NOT true?
- A. T and U may contain some of the same members.
B. There may be a member of T which is not a member of S .
C. A square is always a member of S and T .
D. A rectangle may be a member of set T .
E. NOTA
13. A non-regulation baseball playing field has the shape of a square with playing bases on the vertices. The sides of field are each 90 feet long. The pitcher's mound is 30 feet from home base, on the diagonal which connects home and 2nd base. Find the distance that the pitcher's mound is from 2nd base. Round to the nearest tenth place.
- A. 127.3 B. 125.9
C. 97.3 D. 63.6
E. NOTA
14. Quadrilateral $ABCD$ is inscribed in circle P . If $m\angle A = 20^\circ$, $m\angle B = 2x - 12$ and $m\angle C = 4x - 32$ then $x =$
- A. 48
B. 32
C. 16
D. 13
E. NOTA

Geometry Individual Test

March Regional Competition

15.



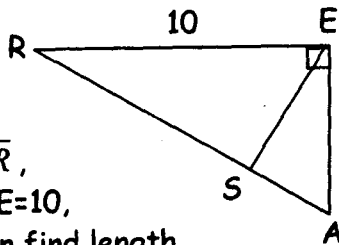
An equilateral triangle ABE shares a side with rectangle $ABCD$ as shown, and E is on side \overline{CD} . If $AB = 10$, then find the perimeter of the rectangle to the nearest tenth place.

- A. 44.5 B. 37.3
C. 28.7 D. 27.1 E. NOTA

16. Two externally tangent circles have radii which measure 12 and 14 cm. A line which is tangent to the smaller circle contains the radius of the larger circle. Find the distance between the centers of the two circles.

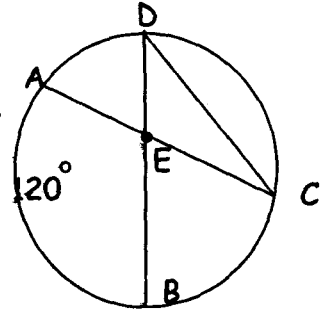
- A. $14\sqrt{3}$ B. $12\sqrt{3}$
C. $14\sqrt{2}$ D. $12\sqrt{2}$
E. NOTA

17. In right $\triangle EAR$, \overline{ES} is the altitude to hypotenuse \overline{AR} , as shown. If $RE = 10$, and $AR = 12$ then find length SA to the nearest hundredth place.



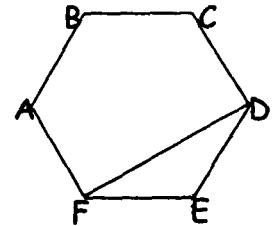
- A. 6.63 B. 4.01
C. 3.67 D. 2.40 E. NOTA

18. In circle R , chords \overline{DC} , \overline{AC} , and \overline{BD} are drawn, with the latter two intersecting at point E . If $m\widehat{CB} = 80^\circ$, $m\widehat{AB} = 120^\circ$ and $m\widehat{CD} = 100^\circ$ then find $m\angle AED$.



- A. 40° B. 60°
C. 70° D. 80° E. NOTA

19. Regular hexagon $ABCDEF$ has sides which are each 10. What is the length of diagonal \overline{FD} ?



- A. $25\sqrt{3}$ B. $15\sqrt{3}$
C. $10\sqrt{3}$ D. 10 E. NOTA

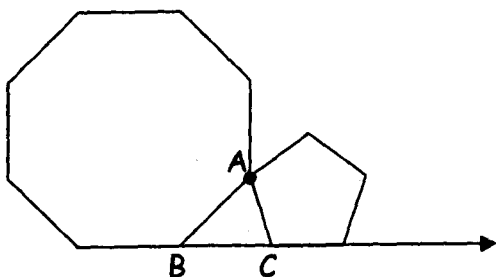
20. An isosceles trapezoid has its shorter base with length 4, legs with lengths 10 (each) and height 8. Find the length of the trapezoid's diagonal, to the nearest tenth place.

- A. $2\sqrt{41}$
B. $8\sqrt{5}$
C. $2\sqrt{65}$
D. 14
E. NOTA

Geometry Individual Test

March Regional Competition

21.



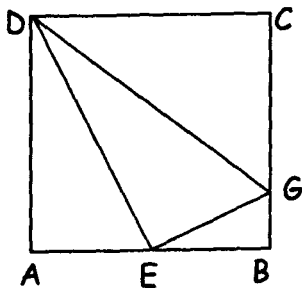
The regular octagon and regular pentagon shown share vertex A and vertices B and C are on the line that contains two of the sides of the polygons as shown. Find $m\angle BAC$.

- A. 65 B. 64
C. 63 D. 62 E. NOTA

22. A rectangle with dimensions 10 cm by 8 cm is formed into a cylinder (with no bases). If bases are then added, the volume of the cylinder to the nearest tenth of a cubic cm could be ____.

- A. 63.7 B. 48.6
C. 16.2 D. 12.7 E. NOTA

23.

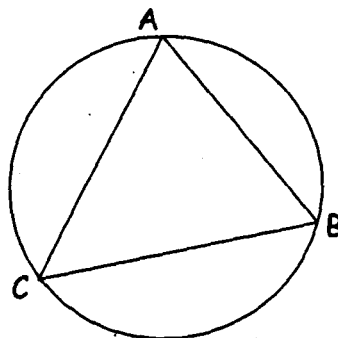


ABCD is a square with $CB=10$. The ratio $BG:GC$ is 1:4. E is the midpoint of \overline{AB} . Give

the ratio of the area of quadrilateral BGDE to the area of square ABCD.

- A. 5:8 B. 3:8
C. 3:5 D. 3:7 E. NOTA

24.



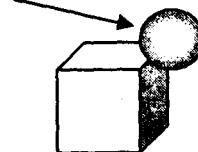
Inscribed triangle ABC has $m\angle A + m\angle B = 140^\circ$ and $m\angle B = 2 \cdot m\widehat{BC}$. If the circle has radius 2, give the length of arc \widehat{BC} .

- A. $\frac{28}{45}\pi$ B. $\frac{14}{5}\pi$
C. $\frac{28}{5}\pi$ D. $\frac{7}{5}\pi$ E. NOTA

25. A regular triangle has area $3\sqrt{3}$. What is its height?

- A. 6 B. $3\sqrt{3}$
C. 3 D. 2 E. NOTA

26. A light is at the corner (vertex) of a building which is a cube with edge 15 ft. A moth will stay within 9 feet of the light and will stay outside the building. That is, he will not travel through the walls. What is the amount of cubic feet of the moth's "domain?" Round to the nearest cubic foot.



not drawn to scale

- A. 5665
B. 3054
C. 2672
D. 2290
E. NOTA

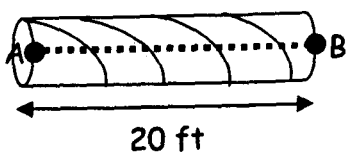
Geometry Individual Test

March Regional Competition

27. A rhombus has diagonals with lengths $2x$ and $2x+2$. If its perimeter is $4x+36$ then $x =$.

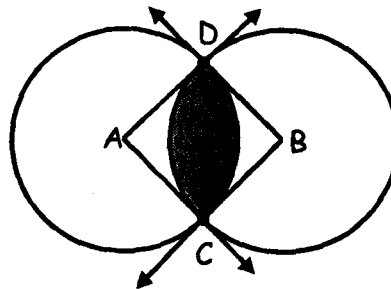
- A. 13 B. 14
C. 20 D. 21
E. NOTA

28. A pole in the shape of a right circular cylinder has ribbon wrapped about it so that the ribbon goes around four times. The ribbon has no slack, and the ends of the ribbon are directly opposite each other on the bases of the cylinder (example: points A and B are "opposite" above.) If the ribbon has length 52 feet and the pole has length 20 feet, then find the circumference of the pole.



- A. 6π B. 12
C. $\frac{12}{\pi}$ D. $\frac{6}{\pi}$ E. NOTA

29.



Congruent circles A and B intersect so that the tangent lines at the points of intersection (C and D) contain the radius of one of the circles. If $AD=6$ then find the area of the shaded region, the intersection of the interiors of the circles.

- A. $9\pi + 4$ B. $36\pi - 18$
C. $18\pi - 36$ D. $18 - 4.5\pi$
E. NOTA

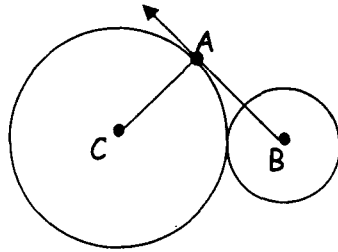
30. A triangle ABC has lengths $AB = 12$ and $AC = 20$. If the triangle is obtuse, then the third side has length x , such that all the possible values of x are given by the conjunction $A < x < B$ or $C < x < D$, and $A < B < C < D$. Give the value of $ABCD$.

- A. $10880\sqrt{2}$
B. $1280\sqrt{514}$
C. $2560\sqrt{594}$
D. $16384\sqrt{34}$
E. NOTA

March Regional

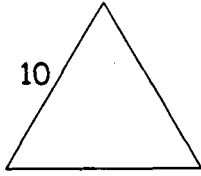
Geometry Team: sponsor's copy

1.

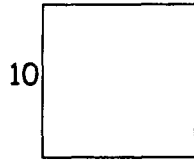


Circle B is externally tangent to circle C. \overline{AB} is tangent to circle C at point A. If $AB=8$ and the radius of circle C is 6, then what is the radius of circle B?

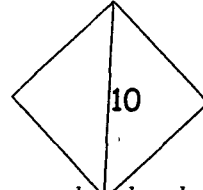
2.



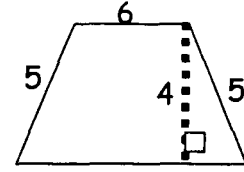
regular triangle
(side = 10)



square
(side = 10)



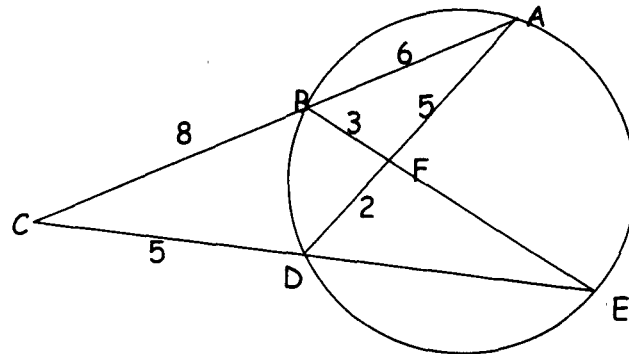
regular rhombus
(diagonal=10)



isosceles trapezoid
(sides as indicated)

The total of the perimeters of all four of the above polygons is $A + B\sqrt{2}$. Find $A + B$.

3.



not drawn
to scale

Secant segments \overline{AC} and \overline{CE} contain points B and D respectively. \overline{AD} and \overline{BE} are chords, as shown, and meet at point F.

$BF=3$, $AF=5$, $AB=6$, $FD=2$, $CB=8$ and $CD=5$. If $ED = \frac{x}{5}$ and $FE = \frac{y}{3}$ then give the value of $x + y$.

4. An arc has a measure of 36 degrees, and a length of $\frac{18}{5}\pi$. Let A be the circumference of the circle of which the arc is a part. Let B be the diameter of the circle of which the arc is a part.

Let C be the area of the circle of which the arc is a part. Find $\frac{C}{AB}$.

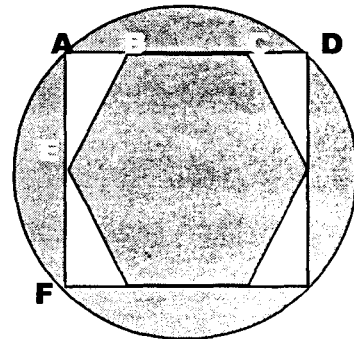
5.

A square with area 100 is inscribed in a circle as shown.

A hexagon is inside the square, and $AB=CD = \frac{1}{2}(BC)$

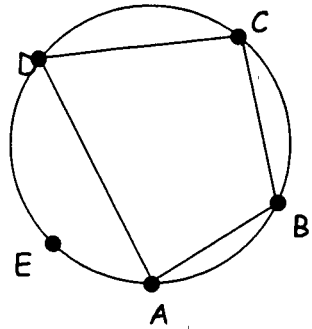
and $AE=EF$. The other sides of the hexagon have the same relationship to the square's sides.

Find the shaded area to the nearest tenth place.



6.

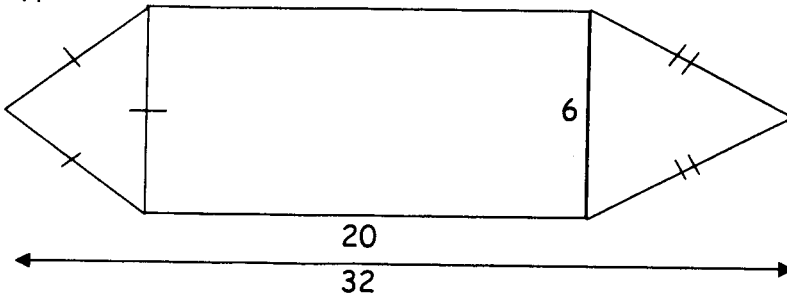
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The degree measure of arcs \widehat{AB} , \widehat{BC} , \widehat{DC} and \widehat{DEA} are y , x , $3x$ and $9y$ respectively. Also, $x + y = 48$

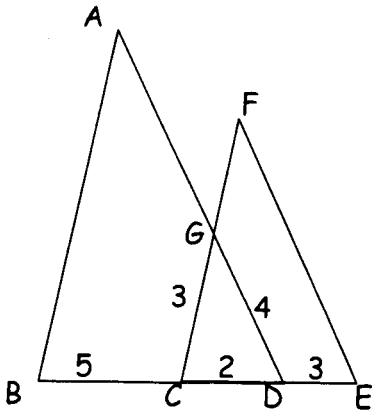
Let R be the angle measure of $\angle CBA$.
Give the sum: $x + 2y + R$.

7.



The figure shown is a rectangle with an equilateral triangle and an isosceles triangle, both sharing a side with the rectangle as shown. The entire length of the figure is 32. Let A be the area of the equilateral triangle. Let B be the length of the leg of the isosceles triangle. Find the value of $A+B$ to the nearest tenth.

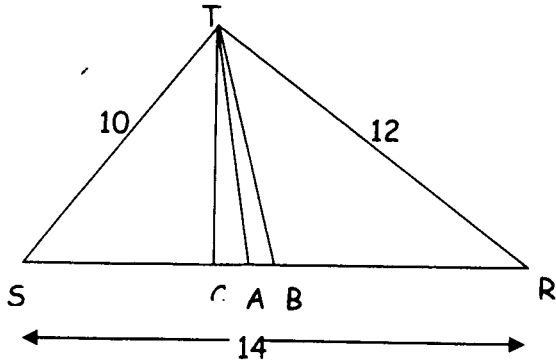
8.



\overline{AB} is parallel to \overline{FC} , and \overline{AD} is parallel to \overline{FE} , with point G on both \overline{FC} and \overline{AD} .
 $BC=5$, $CD=2$, $DE=3$, $GD=4$, $GC=3$.

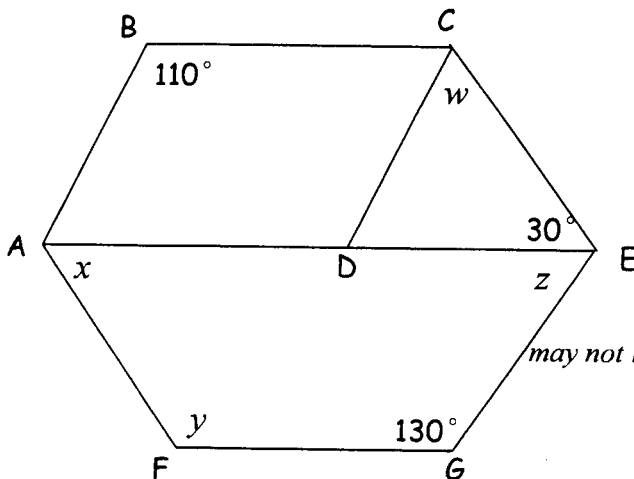
Find the value of
 $AB + AG + FG + (m\angle B - m\angle GCD)$

9.



Triangle SRT is drawn with altitude \overline{TC} , angle bisector \overline{TA} , and median \overline{TB} .
If $TC=8.2$, then find the value of $SC+SA+SB$ to the nearest whole number.

10.

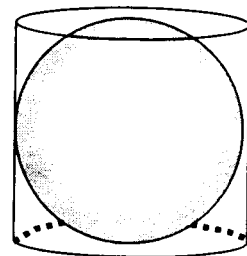


Parallelogram $ABCD$ shares a side with $\triangle DCE$, and trapezoid $AEGF$ has D on its larger base.

If angles of the polygons are labeled as shown with measures x , y , z and w then find $w+x+y+z$.

may not be drawn to scale

11. A spherical ball has radius 6 and is inscribed in a right circular cylinder as shown, so that its sides touch the cylinder's bases, and its great circle is on the cylinder.

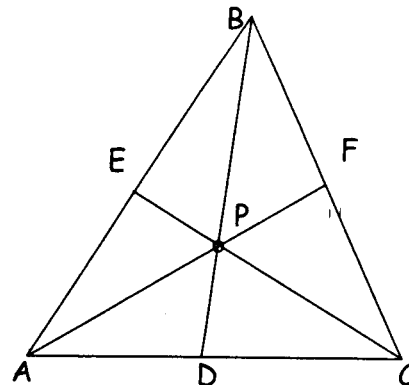


Find the volume of the region outside of the sphere and inside of the cylinder.

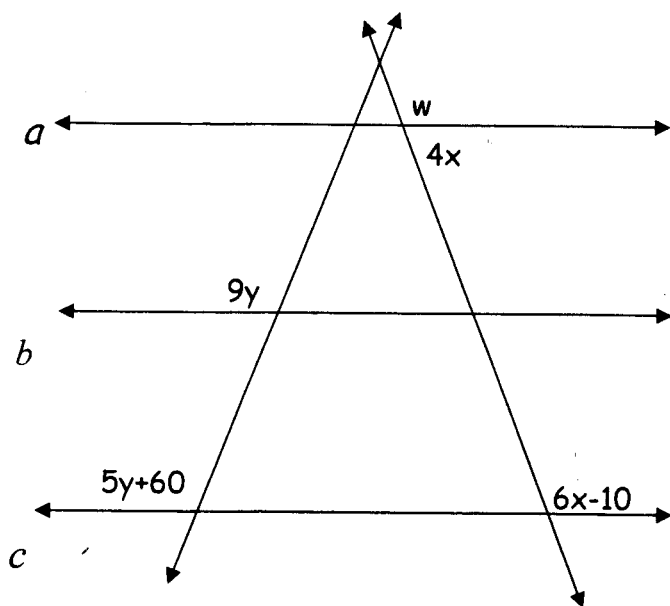
12.

\overline{BD} , \overline{CE} , and \overline{AF} bisect \overline{AC} , \overline{BA} , and \overline{BC} respectively, and all meet at point P.

If $AP=6$, and $BD=12$ then find the value of $AF + PD$.



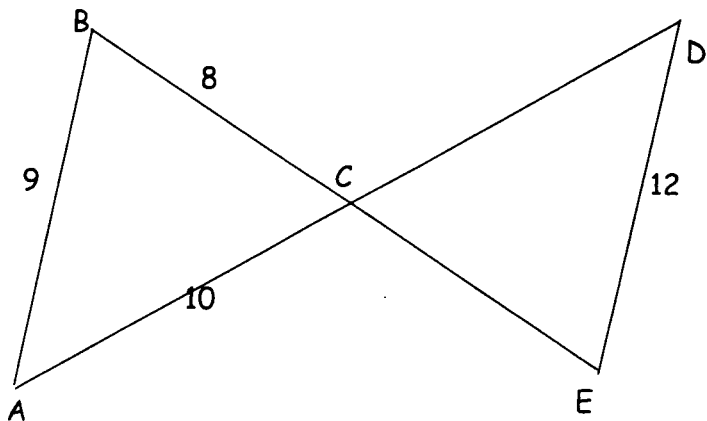
13.



All of the expressions shown are angle measures. If lines a , b , and c are all parallel, then find the measure of the obtuse angle (w).

14. In $\triangle RST$, angle T measures 30 degrees more than angle S. Angle R measures four less than half of angle S. Find $m\angle S$.

15.



$$\overline{BA} \parallel \overline{DE} \quad \text{and}$$

$$BA=9, \quad BC=8, \quad DE=12 \quad \text{and} \quad CA=10.$$

$$CD + CE = ?$$

Answers: GEOMETRY TEAM

1. 4
2. 118
3. 97
4. $\frac{1}{4}$ or 0.25
5. 132.1
6. 232
7. 23.0 or 23
8. 25
9. 19
10. 310
11. 144π
12. 13
13. 121
14. 61.6 or $\frac{308}{5}$
15. 24

Answers: GEOMETRY INDIVIDUAL

1	B	11	D	21	C
2	A	12	B	22	A
3	B	13	C	23	B
4	D	14	A	24	A
5	C	15	B	25	C
6	A	16	E	26	C
7	A	17	C	27	C
8	B	18	C	28	B
9	C	19	C	29	C
10	C	20	A	30	D

- Connect B and C to make a right triangle, with hypotenuse 10. Since the radius of one circle is 6 then the radius of the other circle is $10-6=4$.
- $30+40+20\sqrt{2}+(6+5+5+12)=98+20\sqrt{2}$. $A+B=98+20=118$.
- $8(8+6)=5(5+DE)$ so $DE=87/5$. and x must be 87. $3(FE)=2(5)$ and $FE=10/3$ so $y=10$. The sum is $87+10=97$.
- $\frac{36}{360}(2\pi r)=\frac{18}{5}\pi$ and $r=18$. So $A=\text{circumference}=36\pi$ and B is diameter = 36 and C is area = 324π
 $C/(AB)=\frac{324\pi}{36\pi \cdot 36}=\frac{1}{4}$. The answer is 1/4 or 0.25.
- The diagonal of the square is the diameter of the circle: $10\sqrt{2}$ and the radius is half of this. $r=5\sqrt{2}$. The white triangles are each $1/2$ times 2.5 times 5, and there are four of them. So Circle-4triangles is the shaded area: $50\pi - 25 \approx 132.1$. The answer is 132.1
- The sum of all of the arcs is 360: $4x+10y=360$, divide by 2: $2x+5y=180$. Since $x+y=48$ we solve to get $x=20$ and $y=28$. $m\angle CBA$ is half of $9y+3x$, so $R=156$. $x+2y+R=232$.
- Since the height of the eq. triangle is $3\sqrt{3}$, the height of the other triangle is $32-20-3\sqrt{3}$. A is the area of the eq triangle which is $\frac{1}{4}side^2\sqrt{3}=9\sqrt{3}$ and B is obtained with the Pyth. Theorem:
 $3^2+(12-3\sqrt{3})^2=l^2$, 7.43588. So $A+B$ is approximately 23.0 or 23
- There are three similar triangles in the figure. The small one in the middle GCD , and ABD and FCE .
 Comparing each side to that of the small triangle: $\frac{7}{2}=\frac{4+AG}{4}$, $\frac{2}{7}=\frac{3}{BA}$, $\frac{3+FG}{5}=\frac{3}{2}$ we get
 $AB=10.5$, $AG=10$ and $FG=4.5$. The angles requested are congruent by corresponding angles, and so will subtract to 0. So we add the three sides and get 25.
- Using the Pythagorean Theorem, we get $10^2=8.2^2+(SC)^2$, so $SC=5.72364$ (approx.). For the bisector $\frac{10}{SA}=\frac{12}{14-SA}$ to get $SA=6.3636\dots$ and for SB , it is just half of SR , 7. The sum is then rounded to 19.
- Since the bottom polygon is a trapezoid, z must be 50, due to parallel lines. And $x+y=180$ for the same reason. Angle CDA is 110 since opposite angles of a parallelogram are congruent, and CDE is 70 due to linear pairs. DCE is then 80. $80+180+50=310$.
- The height of the cylinder is $2r$, which is 12. So Volume of the cylinder is $\pi 36(12)=432\pi$. The sphere is $\frac{4}{3}\pi 216=288\pi$ The difference is 144\pi
- The centroid of a triangle meets at a point $2/3$ of the median from a vertex. So $6=(2/3)(AF)$ and $AF=9$. And $(2/3)(12)=8=BP$, and PD must then be $12-8=4$. So the sum of 9 and 4 is 13.
- Due to corresponding angles, $9y=5y+60$, and due to interior angles, same side of the transversal, $4x+6x-10=180$. The little triangle at the top then has bottom interior angles $4x$ and $180-9y$, or 76 and 45. This sum is equal to w , the sum of the remote interior angles of the triangle: 121.
- $(30+S)+(0.5X-4)+S=180$, $2.5S=154$. $S=\underline{61.6 \text{ or } 308/5}$
- $\frac{CD}{10}=\frac{12}{9}$, $\frac{CD}{8}=\frac{12}{9}$ which gives $13.3333+10.6\dots=\underline{24}$.

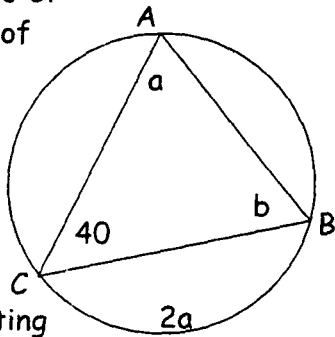
Geometry Individual Test

March Regional Competition

22. A. Either dimension can be the height of the cylinder. So if we use 10 for the circumference of the base, we get the radius is $\frac{5}{\pi}$. Using this to get the volume gives 63.7.

23. B. If we say $BC=10=4x$ then we get $BG=2.5$ and $GC=7.5$. Using the Pyth. Th. to get EG gives $2.5\sqrt{5}$ and likewise $DE=5\sqrt{5}$. The area of quad. $BGDE$ is the sum of DEG and EGB which is 37.5. Putting this over 100, the area of the square, and reducing gives 3:8.

24. A. Take half of arc AB to get angle C is 40. So $a+b=140$. $b=2(2a)$ from the second equation given. Substituting gives $5a=140$ and $a=28$. Double this to get arc BC is 56, and the arc length is $\frac{56}{360}(2\pi r) = \frac{28}{45}\pi$.



25. C. The area of an equilateral triangle is $\frac{side^2}{4}\sqrt{3}$ so we set this equal to $3\sqrt{3}$ to

get the side is $2\sqrt{3}$. The height is then half of this, times $\sqrt{3}$ which gives 3.

26. C. The "domain" of the moth is $\frac{7}{8}$ of the volume of a sphere.

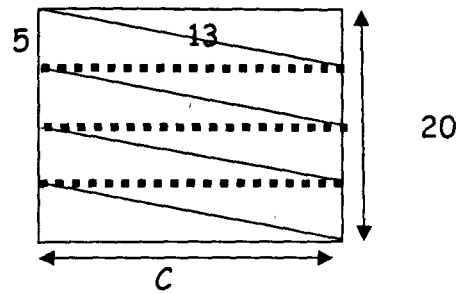
27. C. Taking half of the diagonals we get a small right triangle (since diagonals are perpendicular in a rhombus) with leg lengths x and $(x+1)$. Divide the perimeter by 4 to get side (hypotenuse) $x+9$. Using the Pythagorean Theorem we get

$$x^2 + (x+1)^2 = (x+9)^2$$

$$x^2 + x^2 + 2x + 1 = x^2 + 18x + 81$$

$$x^2 - 16x - 80 = 0; (x-20)(x+4) = 0 \text{ so } x=20.$$

28.



The ribbon divided by 4 is 13. The height divided by 4 is 5. If we open the cylinder then we get the picture above, with C the circumference. Each dotted line is 12, by the Pyth Th. so $C=12$. **Choice B.**

29. One segment of the circle of central angle 90 degrees and radius

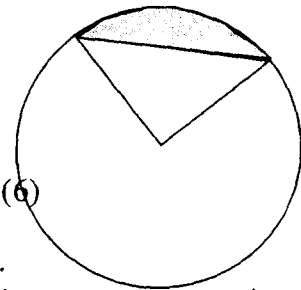
$$6 \text{ is } \frac{90}{360} 36\pi - \frac{1}{2}(6)(6)$$

which is $9\pi - 18\sqrt{3}$.

Double this to get the area requested. Why is the central angle 90 degrees?

Since a radius and a tangent are

perpendicular and we have perpendiculars to the same point on a line we have parallel lines in the picture and $ACBD$ is a parallelogram with a right angle at D . So it is a rectangle. The answer is **choice C.**



30. D. First by the triangle inequality theorem, the third side must be between 8 and 32. Assume that this is a right triangle with legs 20 and 12. We get hypotenuse $\sqrt{544}$ and to have an obtuse triangle we must have that side longer than $\sqrt{544}$. Now assume that the "hypotenuse" is 20, to get the missing side 16. So the side must be shorter than 16. Thus the third side, x , has possibilities:

$8 < x < 16$ or $4\sqrt{34} < x < 32$ The product of all of these is $16384\sqrt{34}$.

Geometry Individual Test

March Regional Competition

Solutions:

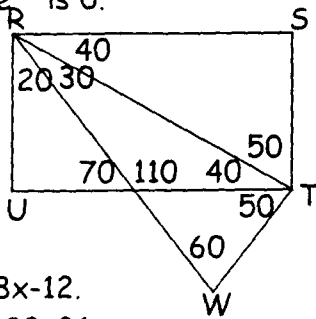
1. B. Base is 12, height is 6, area is $bh=72$.

2. A. The hypotenuse is 10 so its perimeter is 24, and its area is $0.5(6)(8)=24$.

Difference is 0.

3. B.
 $\angle TRX=30$
 $\angle URX=20$

The diff. is 10.



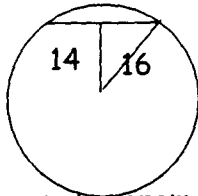
4. D. $3x+3x=8x-12$.

$x=6$, so $x+20=26$.

5. C. Angle T is the vertex angle so $180-(x+20)=160-x$. Divide by 2 to get each base angle, is $80-0.5x$.

6. A. $\frac{x+3}{x} = \frac{14}{10}$. $x=7.5$

7. A. The chord is bisected by the perpendicular from the center,



Using the Pythagorean Theorem, we get the leg of the right triangle to be $\sqrt{60}$ and the chord length double this, approx. 15.5

8. B. In a rectangle the diagonals bisect each other and are congruent, so set any two of these expressions equal to get $x=8$. The diagonal is then $3(8)-10=14$, times 2.

9. C. If we let the long leg be x , then the perimeter is $x+x\sqrt{3}$ and thus this cannot be. If we let the hypotenuse be x , or let the short leg be x then we get the perimeter is $72+24\sqrt{3}$ and then the legs are 24 and $24\sqrt{3}$. The area is then the product of these times $1/2$ which gives choice A, $288\sqrt{3}$.

10. C. The longest chord is the diameter so only choice C is true.

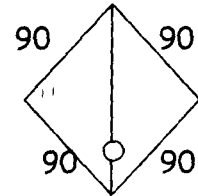
11. D. Since the interior angle and exterior angle add to 180 (linear pair) then we have

$x+x+108=180$ and $x=36$, the ext. angle. So to get the number of sides, we get the total exterior 360 divided by 36 and this gives 10 sides. So $10^2-10=90$.

12. B is the only answer which is not true, since all rhombuses are also parallelograms.

13. C. The diagonal is

$90\sqrt{2}$ and the distance from home base to 2nd is this, less 30. Approx. 97.3.

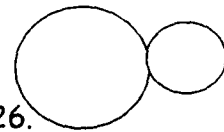


14. A. Opposite angles of a quadrilateral inscribed in a circle are supplementary. So we use angles A and C: $2x-12+4x-32=180$.

That gives $x=48$.

15. B. the height of the triangle is $5\sqrt{3}$ and so the rectangle has perimeter $10+10\sqrt{3}$ approximately 37.3.

16. E. We add the radii and the other information is not needed. The sum is 26.



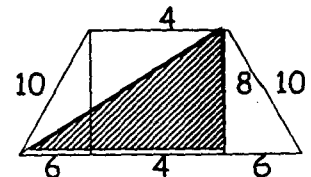
17. C. Using the geometric mean formula, we have $10^2 = RS \cdot 12$; $SA = 12 - RS$ which is approximately 3.67.

18. C. The sum of the arcs is 360 so arc $AD=60$. We get $m\angle AED$ by averaging arcs AD and BC, to get 70.

19. C. A closeup of the triangle is shown. so the missing side is twice $5\sqrt{3}$.



20. A. The heights create two 6-8-10 right triangles. To get the diagonal we use the Pyth. Th. :



$$8^2 + 10^2 = d^2 \quad 2\sqrt{41}$$

21. C. The octagon has angles of 135 and the pentagon has angles of 108. Getting the linear pair angles, and then using the fact that the angles of a triangle add to 180 gives $m\angle BAC=63$.