Algebra I

FELIX VARELA HIGH SCHOOL
FAMAT REGIONAL COMPETITION
FEBRUARY 4, 2006

For all questions, choice e) NOTA, means "None of the above answers is correct."

1) Consider the following sequence: 2, 3, 5, 7, ... What is the sum of the next two terms?
   a) 20
   b) 21
   c) 23
   d) 24
   e) NOTA

2) The sum of three consecutive odd integers is -27. What is their product?
   a) -2145
   b) -1320
   c) -720
   d) -693
   e) NOTA

3) Bryan decides to drink diluted Powerade during Mu Alpha Theta practice. He has a mixture that is 60% Powerade, but it is too strong. How many cups of pure water and how many cups of the old mixture should he use to make a new 8-cup mixture with a refreshing 55% water content?
   a) 2 cups of pure water, 6 cups of the 60% Powerade mixture
   b) 3 cups of pure water, 5 cups of the 60% Powerade mixture
   c) 5 cups of pure water, 3 cups of the 60% Powerade mixture
   d) 6 cups of pure water, 2 cups of the 60% Powerade mixture
   e) NOTA

4) Which of the following describes the number zero?
   i. rational number   ii. whole number   iii. natural number
   a) i & ii only
   b) ii & iii only
   c) i & iii only
   d) i, ii, & iii
   e) NOTA

5) Find the sum of the factors of 2006.
   a) 1006
   b) 1234
   c) 3012
   d) 3239
   e) NOTA

6) What is the absolute value of the difference between the slope and the y-intercept of the line through the point (1, -1), perpendicular to the line $3x - 15y = -28$?
   a) -9
   b) -1
   c) 1
   d) 9
   e) NOTA

7) What is the least common multiple of 9, 10, 11, 12, 13, and 14?
   a) 360360
   b) 540540
   c) 720720
   d) 2162160
   e) NOTA

8) Fifty-one more than nine times a nonnegative number n is less than eighty-seven plus six times the number. What is the set of all possible such n?

a) $\{n \mid n < 4\}$
b) $\{n \mid n < 12\}$
c) $\{n \mid 0 < n < 4\}$
d) $\{n \mid 0 < n < 12\}$
e) NOTA

9) Anthony, a poor college student, has a mere $9.55 in his pocket. The sum is made up of 52 coins, all either dimes or quarters. How many dimes does he have?

a) 23
b) 25
c) 27
d) 29
e) NOTA

10) Please evaluate $\sqrt[3]{(-4)^{12}}$.

a) No real answer
b) -256
c) -64
d) 16777216
e) NOTA

11) One google is the number with a one followed by one hundred zeros. One googleplex is the number one followed by one google zeroes. Please write one googleplex in scientific notation.

a) 1^{100}
b) 1.0×10^{100}
c) 1.0×10^{200}
d) $1.0 \times 10^{10^{100}}$
e) NOTA

12) The polynomial $84mp + 147mq - 100np - 175nq$ can be factored into the form $(am + bn)(cp + dq)$ where a, b, c, and d are integers and a is positive. What is $a + b + c + d$?

a) 7
b) 15
c) 57
d) 65
e) NOTA

13) Find the sum of the entries in the matrix resulting from $\begin{bmatrix} -5 & -11 \\ 54 & 1 \end{bmatrix} - \begin{bmatrix} 9 & 13 \\ -3 & 6 \end{bmatrix}$

a) -24
b) -5
c) 14
d) 64
e) NOTA

14) Please simplify the following: $\left[\dfrac{\dfrac{30x^2-30y^2}{42x^2y} \div \dfrac{yx+y^2}{2x+3y}}{10x+15y}\right]^{-1}$

a) $\dfrac{x-y}{7x^2y^2}$

b) $\dfrac{28x^4+84x^3y+63x^2y^2}{x^3+x^2y-xy^2-y^3}$

c) $\dfrac{7x^2y^2}{x-y}$

d) $\dfrac{x^3+x^2y-xy^2-y^3}{28x^4+84x^3y+63x^2y^2}$

e) NOTA

15) If the operation ☼ is defined such that $a☼b = -a^b + 2ab - b^a$ for all real numbers a and b, then what is the value of $[3☼(1☼2)]^4$?

a) 0
b) 2
c) 16
d) 65536
e) NOTA

16) The perimeter of a rectangle with integer dimensions is 16. Let l be the length of the rectangle and let w be the width. Which of the following cannot be the value of lw?

a) 7
b) 12
c) 15
d) 16
e) NOTA

17) Solve $5x+5m = xy-4n$ for x in terms of m, n, and y.

a) $\frac{5m-4n}{y-5}$
b) $\frac{5m+4n}{y-5}$
c) $\frac{5m-4n}{y+5}$
d) $\frac{5m+4n}{y+5}$
e) NOTA

18) Shira, a new driver, is late to a Mu Alpha Theta competition. She wants to get there as quickly as possible, so she wants to drive as quickly as she can safely and legally do so. Thus, she maintains a speed no more than 4 mph above or below the posted speed limit. If the posted speed limit is 45 mph, what is the sum of all possible integer values of her desired speed in miles per hour?

a) 315
b) 356
c) 364
d) 405
e) NOTA

19) When $19x^3 - 38x^2 - 152x$ is factored, it takes the form $Ax(x+B)(x+C)$ where $A, B,$ and C are real numbers and A is positive. Please find $A+B+C$.
a) 13
b) 17
c) 21
d) 25
e) NOTA

20) Which of the following properties excludes the number zero?
a) Additive Identity Property
b) Multiplicative Identity Property
c) Multiplicative Inverse Property
d) All of the Above
e) NOTA

21) Which of the following is the set of all integers less than 7 units away from the number five and at least two units away from the number 3?
a) $(-2,12)$
b) $\{-1,0,1,...,9,10,11\}$
c) $\{-1,0,1\} \cup \{5,6,7,...\}$
d) $\{-1,0,1\} \cup \{5,6,7,8,9,10,11\}$
e) NOTA

22) Solving $29y^2 - 3y - 10 = 0$ for y yields two answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are all positive real numbers. Please find $a+b+c$.
a) 1195
b) 1201
c) 1224
d) 1230
e) NOTA

23) Consider the line $2x + 3y + 6 = 0$. What is the product of the x-intercept and the y-intercept?
a) -6
b) -5
c) 0
d) 6
e) NOTA

24) Romeo and Juliet had a fight. Juliet got into her car and sped away at 60 mph. Eighteen-hundred seconds later, Romeo started to follow her. He went 70 mph in the same direction. If Juliet left at 11:00 a.m. on a straight road, what time will Romeo catch up to her?
a) 2:00 p.m.
b) 2:30 p.m.
c) 3:00 p.m.
d) 3:30 p.m.
e) NOTA

25) The sum of two numbers is 17; their product is 60. What is the sum of their squares?
a) 136
b) 157
c) 169
d) 241
e) NOTA

26) The midpoint between $\left(\frac{2}{3},3\right)$ and (x,y) is $\left(1,\frac{1}{2}\right)$. What is the value of $21xy$?

a) -56
b) 0
c) $\frac{35}{8}$
d) $\frac{245}{8}$
e) NOTA

27) Please simplify the following expression, eliminating negative exponents: $\dfrac{5\left(7m^2np^{-1}\right)^5}{343\left(n^6m^{-4}\right)^{-2}p^{11}}$

a) $\dfrac{5m^{18}n^{17}}{343p^{16}}$
b) $\dfrac{5m^{18}}{49n^7p^6}$
c) $\dfrac{5m^2}{7n^7p^6}$
d) $\dfrac{245m^2n^{17}}{p^{16}}$
e) NOTA

28) The tens digit of a certain two-digit positive number is equal to half its units digit. In addition, reversing the digits of the number yields a new value equal to thrice the original value less thirty. What is one-half of the original number?

a) 12
b) 21
c) 24
d) 42
e) NOTA

29) Elizabeth went shopping at a store where all items were 70% off. She bought an item with a coupon for 10% off the discounted price. The item cost $5 at regular price, so Elizabeth handed the cashier a five dollar bill. Assuming that the transactions are rounded to the nearest penny and there is no tax, how much change did Elizabeth receive?

a) $1.00
b) $1.35
c) $3.65
d) $4.00
e) NOTA

30) If $0.3\overline{8}$ is written as a fraction in lowest terms, what is the sum of the numerator and the denominator?

a) 11
b) 12.5
c) 25
d) 125
e) NOTA

Solutions Algebra 1 Varela Feb 4, 2006

1)The sequence represents prime numbers arranged in ascending order. The fifth and sixth prime numbers are 11 and $13. 11+13=24$, choice D.
2) Let the numbers be $x, x+2,$ and $x+4$. Now, $x+(x+2)+(x+4)=-27$, so $3x+6=-27$, so $x=-11$. Thus, the numbers are -11, -9, and -7; their product is -693, choice D.
3) Let w be the amount of pure water he should add. If the new mixture is 8 cups, he needs $8-w$ cups of 60% Powerade, which, thus contains 40% $=0.4(8-w)$ cups of water. The new mixture needs 55% water = (0.55)(8 cups) = 4.4 cups. Hence, $4.4=w+0.4(8-w)$, and w = 2 cups. Therefore, the mixture needs 6 cups of 60% Powerade, A.
4) Zero is rational and a whole number, but not a natural number. Only i and ii are correct: choice A.
5) $2006=2\times17\times59$, so the factors of 2006 are 1, 2, 17, 34, 59, 118, 1003, and 2006. Their sum is $1+2+17+4+59+118+1003+2006=3240$, which is E, NOTA.
6) The line is perpendicular to $3x-15y=-28$; its slope is -5. Its equations in point-slope and slope-intercept forms are $y-(-1)=-5(x-1)$ and $y=-5x+4$. Lastly, $|-5-4|=9$, choice D.
7) Write the prime factorizations of each of the numbers from 9 to 14: $9=3^2$, $10=2^1*5^1$, $11=11^1$, $12=2^2*3$, $13=13^1$, $14=2^1*7^1$. Multiply the highest powers of all of the included prime factors: $2^2*3^2*5^1*7^1*11^1*13^1=180180$, choice E.
8) Solving $51+9n<87+6n$ yields $n<12$. The restriction over nonnegative real numbers means $n\geq0$. Combing the two inequalities yields $0\leq n<12$, which is choice E, NOTA.
9) Let d be the number of dimes and q the number of quarters. There are 52 coins, so $d+q=52$. Since dimes are worth \$0.10 each and quarters \$0.25, $0.10d+0.25q=9.55$. The solution is $d=23, q=29$. There are 23 dimes, choice A.
10) Solve it step by step. $\sqrt[3]{(-4)^{12}}=(-4)^{\frac{12}{3}}=(-4)^4=256$. E, NOTA.
11) One google is followed by 100 zeros, so it is written in scientific notation as 1.0×10^{100}, or simply as 10^{100}. A googleplex is followed by a google zeros, so it is multiplied by ten raised to the one google power, or $1.0\times10^{10^{100}}$, choice D.
12) Factor by grouping. $84mp+147mq-100np-175nq=21m(4p+7q)-25n(4p+7q)$ $=(21m-25n)(4p+7q)$. Thus, $a+b+c+d=21+(-25)+4+7=7$, choice A.
13) Subtracting the entries in the second matrix from the entries in the first matrix yields $\begin{bmatrix}-14 & -24\\ 57 & -5\end{bmatrix}$. The sum of its entries is $-14+(-24)+57+(-5)=14$, choice C.
14) First eliminate the outer exponent. Change $\left[\dfrac{\frac{30x^2-30y^2}{42x^2y}\div\frac{yx+y^2}{2x+3y}}{10x+15y}\right]^{-1}$ to $\dfrac{10x+15y}{\frac{30x^2-30y^2}{42x^2y}\div\frac{yx+y^2}{2x+3y}}$. Then simplify the fractions further: $(10x+15y)\div(\frac{30x^2-30y^2}{42x^2y}\div\frac{yx+y^2}{2x+3y})=(10x+15y)\div(\frac{30x^2-30y^2}{42x^2y}\times\frac{2x+3y}{yx+y^2})=$
$(10x+15y)\div(\frac{6(5)(x+y)(x-y)}{6(7)x^2y}\times\frac{2x+3y}{y(x+y)})=(10x+15y)\div(\frac{5(x-y)(2x+3y)}{7x^2y})=(10x+15y)\times(\frac{7x^2y}{5(x-y)(2x+3y)})=\frac{7x^2y^2}{x-y}$, C.
15) First evaluate 1☼2 : 1☼2 = $-1^2+2(1)(2)-2^1=1$. Next evaluate 3☼1: 3☼1 = $-3^1+2(3)(1)-1^3=2$. Finally, $[3☼(1☼2)]^4=[3☼1]^4=2^4=16$, which is choice C.
16) If the perimeter is 16, then $l+w=8$. Furthermore, the side lengths must be integers, so l and w can be 1 and 7, 2 and 6, 3 and 5, or 4 and 4. Those products are 7, 12, 15, and 16. Thus, all four choices can be the value of lw, and by default, the answer is E, NOTA.
17) Treat all variables other than x as constants. Then, group the x's on one side and the constants on the other side: $5m+4n=xy-5x$. Next, factor out an x from the right side: $5m+4n=x(y-5)$. Finally, divide both sides by $y-5$. The answer is $x=\frac{5m+4n}{y-5}$, choice B.
18) Let s be her speed in mph. $|s-45|\leq4$, so $41\leq s\leq49$. Therefore, the sum of all possible integer values of her speed is 41+42+43+...+47+48+49 = 405, choice D.

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19) All 3 terms are divisible by 19*x*. $19x^3 - 38x^2 - 152x = 19x(x^2 - 2x - 8) = 19x(x-4)(x+2)$. As a result, $A + B + C = 19 + (-4) + 2 = 17$, choice B.

20) The only one that excludes zero is C, the multiplicative inverse property, which states "For every nonzero number $\frac{a}{b}$, where *a* and *b* are nonzero, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \times \frac{b}{a} = 1$.

21) This problem requires two inequalities. Using the variable *n*, the first inequality is $|n-5| < 7$, which simplifies to $-7 < n - 5 < 7$ and $-2 < n < 12$. The second inequality is $|n-3| \geq 2$, which simplifies to $n - 3 \geq 2$ or $n - 3 \leq -2$, and further to $n \geq 5$ or $n \leq 1$. The solution must satisfy all of these conditions: $-2 < n < 12$ and $n \geq 5$ or $n \leq 1$. Since it is restricted to integers, the final answer is $\{-1, 0, 1\} \text{U} \{5, 6, 7, 8, 9, 10, 11\}$, D.

22) Using the quadratic formula to solve for *y* yields y = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(29)(-10)}}{2(29)} = \frac{3 \pm \sqrt{1169}}{58}$. That means $a + b + c = 3 + 1169 + 58 = 1230$, choice D.

23) Subtract 6 from both sides so that the equation is in standard form: $2x + 3y = -6$. Substituting zero in for *y* yields an *x*-intercept of -3 and substituting zero in for *x* yields a *y*-intercept of -2. Their product is (-2)(-3) = 6, choice D.

24) Let *t* be the time in hours since Juliet left. At any given time, Juliet is at a distance of $d = 60t$. Romeo leaves 1800 seconds, or one-half hour later, so he has traveled one-half an hour less time than her. His equation is $d = 70(t - 0.5)$. Set these equations equal and solve for *t*. The result is $t = 3.5$. Romeo catches up to Juliet 3.5 hours after 11:00 a.m., or at 2:30 p.m, choice B.

25) Set up two equations, $x + y = 17$, and $xy = 60$. Square both sides of the first equation, and get $(x+y)^2 = x^2 + 2xy + y^2 = 17^2 = 289$. From the second equation, $xy = 60$, so $2xy = 120$. This means $x^2 + 2xy + y^2 = x^2 + 120 + y^2 = 289$. Subtract 120 and get $x^2 + y^2 = 169$, choice C.

26) Set up the following equations and solve for *x* and *y*: $\frac{\frac{2}{3} + x}{2} = 1$ and $\frac{3 + y}{2} = \frac{1}{2}$. The result is $(\frac{4}{3}, -2)$. The value of $21xy$ is $21(\frac{4}{3})(-2) = -56$, choice A.

27) First, distribute the exponents: $\frac{5(7m^2np^{-1})^5}{343(n^6m^{-4})^{-2}p^{11}} = \frac{84035m^{10}n^5p^{-5}}{343n^{-12}m^8p^{11}}$. Then treat the constant and each variable separately, and reduce. $\frac{84035}{343} = 245$, $\frac{m^{10}}{m^8} = m^2$, $\frac{n^5}{n^{-12}} = n^{17}$, and $\frac{p^{-5}}{p^{11}} = \frac{1}{p^{16}}$. Put it all together: $\frac{245m^2n^{17}}{p^{16}}$, D.

28) Denote *t* as the tens digit and *u* as the units digit. The original number is $10t + u$. From the question, one can obtain a system of equations: $t = \frac{1}{2}u$ and $10u + t = 3(10t + u) - 30$. The solution is $t = 2$ and $u = 4$. Therefore, the original number is 24 and half of it is 12, choice A.

29) She had a 70% discount, so her discounted price was 30% of $5.00, or $1.50. She received 10% off of that price, so she ended up paying 90% of $1.50, or $1.35. Her change from $5 was $3.65, C.

30) Let $n = 0.3\overline{8}$. Multiply it by 10 and it will still repeat, as such: $10n = 3.8\overline{8}$. Now, subtract the first equation from the second to cancel the repeating portion of the decimal. The result is $9n = 3.5$. Finally, $n = \frac{3.5}{9} = \frac{35}{90} = \frac{7}{18}$. The sum of the numerator and the denominator is 7 + 18 = 25, choice C.

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Algebra I Individual
Answers

1) D
2) D
3) A
4) A
5) E (3240)
6) D
7) E (180180)
8) E ($0 \leq n < 12$)
9) A
10) E (256)
11) D
12) A
13) C
14) C
15) C
16) E (all are possible values)
17) B
18) D
19) B
20) C
21) D
22) D
23) D
24) B
25) C
26) A
27) D
28) A
29) C
30) C

February 2006 Algebra I Team

1) A. What is the greatest common factor of $2x^2 - x - 91$ and $4x^2 - 19x - 63$?

B. What is the least common multiple of $3x^2 - 23x + 14$ and $x^2 - 4x - 21$?

C. What is the conjugate of $3x + 2$?

D. What is the sum of all values of k for which $4x^2 + kx + 1$ is a perfect square?

Please express $\frac{B+D}{AC}$ in terms of x.

2) Evaluate the truth or falsity of the following statements. Add the numbers in parentheses next to each true statement. If the statement is false, do nothing – the value for that statement is zero. Give the resulting sum. (If there are no true statements, then the resulting sum is zero.)

If a is an integer, then $-a^2$ must be negative. (-1)

All rational numbers have a reciprocal. (8)

Addition and multiplication are commutative, associative operations. (-27)

Multiplication distributes over subtraction. (64)

The set of positive integers is closed under division. (-125)

The product of a number and its additive inverse is always nonnegative. (216)

The square root of the square of a number always equals the number itself. (-343)

A number always equals itself. (512)

3) A. Consider the equation $5.1d + b + c - 30ad = 0$. Let $a = 0.17$. If b and c are nonzero constants and the equation is an identity, find the value of b in terms of c. What is the value of b/c ?

B. How many values of x satisfy the following equation: $\frac{62}{15}x + 1 = \frac{1}{5} + 4x + \frac{2}{15}x$?

C. How many solutions does $47(13x - 18) = 66 - 44x$ have?

What is $(A + B)^C$?

4) A. What is 65% of ½ of 360?

B. What percent of 310 is 96.1?

C. $\frac{3}{10}$ is 20% of what number?

D. Solve the following proportion: $\frac{-42}{m} = \frac{12}{16-m}$.

Please find $A - B + C - D$.

5) Mark is taking a physics class for which he has two parts: a lecture and a lab. 25% of his overall grade comes from the lab; 75% comes from the lecture. In his lecture class, each of 3 tests is worth 20% of his grade, homework is worth 15%, and the final exam is 25%. In his lab course, 5 quizzes are collectively worth 20% of his lab grade and the rest of the lab grade will come equally from his 8 lab reports. Suppose in his lecture he has a homework average of 95 and he earned grades of 87, 98, and 94 on his tests. In his lab, he earned grades of 83, 96, 97, 95 and 99 on the quizzes and 86, 90, 91, 93, 95, 97, 100, and 100 on his lab reports. (All grades are out of 100.)

What is Mark's percentage in the lab component alone?

6) Rachel and Jordan standing next to each other on one side of a circular lake and want to travel to the other side. Rachel swims directly across the lake with radius 1980 ft at 264 feet per minute. Jordan runs at 352 feet per minute on the 7040 feet trail around the lake and ends at the same destination point.

A. How many minutes does it take Rachel to cross the lake?

B. How many minutes does it take Jordan to run around the lake?

C. At what speed, in feet per minute, should Jordan have run if he wanted to end at the same time as Rachel?

D. At what speed, in feet per minute, should Rachel have swum if she wanted to cross the lake in 12 minutes?

Please find $\frac{BD}{AC}$.

7) A. What is the reciprocal of the slope of the line perpendicular to $0.1y = \frac{11}{113}x + 7.45$?

B. What is the slope of the line parallel to $5.71x - 3.39y = 42$?

C. What is the slope of the x-axis of a Cartesian Coordinate Plane?

D. What is the value of k such that the line through (2, 3) and (4, k) has a slope of $-k$.

Please simplify $\frac{C-AD}{B}$ into a fraction in lowest terms.

8) Jeff is currently sixteen years old and 165 centimeters tall.

A. If his bed must be at least 5 cm longer than he is tall, what is the set of all possible lengths in centimeters of his bed?

B. If he wants to date women that are shorter than him by at least 2.5 cm but no more than 15 cm, what is the range of heights in centimeters of women he should date?

C. If he grows 1.25 cm per year for the next 4 to 8 years inclusive and then stops, what is the range of his height in centimeters after he stops growing?

D. When he grows old and frail, suppose he will shrink to within 5 cm of his height at age sixteen. What is the range of his decreased height in centimeters?

Please find $(((A \cap C) \cup B) \cup D)$.

9) Consider the following systems of equations. Please find $AB + CD$.

$y = \frac{4}{5}x + 7$	$4y = 4.5x - 148$	$-50y = 79 + 12x$	$17x + 41y = 3$
$2x + 2.8 = 2.5y$	$1.125x - y = 37$	$25x = 6y - 78$	$205y = 85x + 15$

A. How many of the systems are consistent?

B. How many of the systems are dependent?

C. How many are inconsistent?

D. How many are independent?

10) A. Please find the degree of the following monomial $\frac{1}{18}z$?

B. Please find the degree of the following monomial $2xy$?

C. Please find the degree of the following polynomial $\pi m^5 + n^2$.

D. Please find the degree of the following polynomial $7^2a^2b^2 + c^3$.

Now, using the above answers consider the polynomial $x^B - Dx^A - C$. Please factor it.

11) A. Find the discriminant of $7x^2 - 31x + 341 = 0$.

B. How many distinct solutions exist for the equation $x^2 - 23x + 102$?

C. How many rational solutions exist for the equation $323x^2 - 2x - 1$?

D. What is the sum of the distinct solutions to the equation $961x^2 + 186x + 9$?

Please find $AD - BC$.

12) Simplify:

$\begin{pmatrix} \sqrt{50} & \sqrt{32} \\ 7\sqrt{3} & \sqrt{26} \end{pmatrix} + \sqrt{2}\begin{pmatrix} 3 & \sqrt{1} \\ \frac{12}{\sqrt{6}} & \sqrt{13} \end{pmatrix}$ The answer will be in the form $\begin{pmatrix} A\sqrt{B} & C\sqrt{D} \\ E\sqrt{F} & G\sqrt{H} \end{pmatrix}$.

Please find $A - B + C - D + E - F + G - H$.

13) The following are operations used for evaluating numerical expressions. Using the order of operations, rank them from first to last, numbering them from 1 to 4. Then, multiply the rank by the number in parenthesis next to the corresponding operation. Finally, please add all four products.

Addition/Subtraction (1)

Exponentiation (4)

Multiplication/Division (16)

Parentheses (25)

14)A. Find the result of the following expression $\frac{\frac{18}{5}}{\frac{2}{7}} + 79(-3) - 7.2 \times \frac{1}{2}$

B. Find the additive inverse of the result of the following expression $\frac{-3}{4}\left[-6 + \left(-14\right)\right] - 217 \div \left(-31\right)$.

C. Find the multiplicative inverse (in fractional form) of the result of the following expression $17.4\left(-6\right) + 101 - 11.4 \div 19$.

D. Find the result of the following expression $\left|-\left|42 - 3\left(-11\right) - 20 \div \frac{1}{4}\right|\right|$.

Please find $AC - BD$.

15) Consider the points $J(4,1)$, $K(9,3)$, $L(13, -2)$ and $M(3, -6)$.

A. Find the sum of the coefficients of the equation of the line written in standard form containing points J and L.

B. Find the sum of the coefficients of the line written in standard form containing the points K and M.

C. Find the sum of the coordinates of the point of intersection of line containing the points J and L and the line containing the points K and M.

Please find $\left(\frac{B}{A}\right)^C$.

Algebra 1 Team 2/4/06

Solutions

1) A. $2x^2 - x - 91 = (2x+13)(x-7)$, and $4x^2 - 19x - 63 = (4x+9)(x-7)$. Their GCF is $x - 7$.
B. $3x^2 - 23x + 14 = (3x-2)(x-7)$ and $x^2 - 4x - 21 = (x-7)(x+3)$. Their LCM is $(3x-2)(x-7)(x+3)$.
C. The conjugate of $3x + 2$ is $3x - 2$.
D. k = 4, or -4. Their sum is zero.
$$\frac{BD}{AC} = \frac{(3x-2)(x-7)(x+3)+(0)}{(x-7)(3x-2)} = x+3$$

2) The first statement is false, because the number could be zero, and the opposite of its square is zero, which is not a negative number.
The second statement is false because the number zero has no reciprocal.
The third statement is true. Addition and multiplication are commutative (the order in which two numbers are added or multiplied together does not matter), and associative (the order in which three numbers are grouped together and then evaluated does not matter.)
The fourth statement is true. The difference of two numbers is multiplied by a third number is equal to the difference of the product of the first number in the difference and the third number and the product of the second number in the difference and the third number.
The fifth statement is false. Dividing two positive integers does not necessarily yield another positive integer. For example, $2 \div 3 = \frac{2}{3}$, which is not an integer.
The sixth statement is false. Let a be any real number; $-a$ is its additive inverse. Their product is $-a^2$ which is negative unless a is zero.
The seventh statement is false. If the number is negative, then the square of the number is positive and its square root is again positive, so it cannot equal the original number.
The eighth statement is true by the reflexive property of equality.
The desired sum is -27 + 64 + 512 = 549.

3) A. First substitute 0.17 for a, resulting in $5.1d + b + c - 5.1d = b + c = 0$. If this is an identity, then it is always true. Thus, the left member must equal the right, 0, so $b = -c$. Hence, $b/c = -c/c = -1$. Note: b and c are given to be nonzero.
B. The right member simplifies to $\frac{62}{15}x + \frac{1}{5}$; subtract $\frac{62}{15}x$ from both sides and the resulting equation is $1 = \frac{1}{5}$, which clearly has zero solutions.
C. First distribute the 47: $47(13x - 18) = 611x - 846 = 66 - 44x$. Then, group the variable on one side and the constants on the other: $655x = 912$. At this point it is clear that the equation has exactly one solution, and that solution is $\frac{912}{655}$.
$(A+B)^C = (-1+0)^1 = -1$.

4) A. 65% of ½ of 360 is 65% of 180, which is 0.65(180) =117.
B. Set up the following proportion: $\frac{96.1}{310} = \frac{x}{100}$. The solution is x = 31.
C. Set up the following proportion: $\frac{\frac{3}{10}}{x} = \frac{20}{100}$. The solution is x = 1.5
D. Cross multiplying yields the equation $-672 + 42m = 12m$; m = 22.4
$A - B + C - D = 117 - 31 + 1.5 - 22.4 = 65.1$

Solutions Algebra I Team 2/4/01

5) A. First, find Mark's quiz average in the lab course. It is (88 + 96 + 97 + 95 + 99)/4 = 94. Second, find his lab report average: (86 + 90 + 91 + 93 + 95 + 97 + 100 + 100)/8 = 94. Finally, his grade in the lab is 0.20(94) + 0.80(94) = 94.

6) A. Rachel swims a total of 3960 ft at 264 ft/min. Use the formula d=rt. It takes t = d/r = 3960 ft/264 ft/min = 15 minutes.
B. Similarly, Jordan takes t = d/r = 7040 ft/352 ft/min = 20 minutes to run around the lake.
C. Jordan now wants to make it in the same time as Rachel, 15 minutes. This means his speed will have to be r = d/t = 7040 ft/15 minutes = 1408/3 ft/min.
D. Similarly, Rachel's speed will have to be r = d/t = (3960 ft/12 min) = 330 ft/min.
BD/AC = 20(330)/[15(1408/3)]=15/16.

7) A. The line has a slope of $\frac{110}{113}$. A line perpendicular to that has a slope that is its opposite reciprocal, $\frac{-113}{110}$. The reciprocal of that slope is $\frac{-110}{113}$.
B. The slope of the line $5.71x - 3.39y = 42$ is $\frac{571}{339}$. A line parallel to that has the same slope.
C. The x-axis is a horizontal line, and as such, has a slope of zero.
D. Use the equation $\frac{k-3}{4-2} = -k$, and solve for k. The solution is $k = 1$.
(C-AD)/B = $(0 - (\frac{-110}{113})(1))/\frac{571}{339} = \frac{330}{571}$.

8) A. Jeff is 165 cm tall. Let b denote the length of his bed. The bed must be at least 5 cm longer, so $b - 165 \geq 5$, and $b \geq 170$. Thus, the answer is $[170, \infty)$
B. Let h denote the height of the woman. He wants to be at least 2.5 cm taller, so $165 - h \geq 2.5$, and $h \leq 162.5$, but no more than 15 cm, so $165 - h \leq 15$ and $h \geq 150$. The result: [150, 162.5].
C. If he only grows for 4 years, he will grow 5 cm. If he grows for the full 8 years, he will grow 10 cm. Thus, he will grow by 5 to 10 inchese, so his height will be between 170 and 175, inclusive: [170, 175].
D. His height at 16 is 165 cm, and his height will be within 5 cm of that. Thus his new height is between 160 and 170 inches, inclusive: [160, 170]
$(((A \cap C) \cup B) \cup D) = (([170, \infty) \cap [170, 175]) \cup [150, 162.5]) \cup [160, 170] = ([170, 175] \cup [150, 162.5]) \cup [160, 170] = [150, 175]$.

9) There is no need to solve any of the systems, although the problem can be done in that manner. An easier way to answer the questions is to compare the slopes and y-intercepts of the lines in each system. The first system had lines with the same slopes but different intercepts; the lines are parallel, and the system has no solution. It is inconsistent. The second system had lines with the same slopes and the same intercepts; the lines are coincident, and there are infinitely many solutions. This system is consistent and dependent. The third system and fourth systems had lines with different slopes; both have exactly one solution. They are consistent and independent. Thus, the answers to A through D are 3, 1, 1, and 2 respectively, and the final answer is 5.

10) The degree of a monomial is the sum of the exponents of its variables. The degree of a polynomial is the greatest of the degrees of its terms. Thus, $\frac{1}{18}z$ has degree 1, $2xy$ has degree $1 + 1 = 2$, $\pi m^2 + n^5$ has degree 5, because 5 is greater than 2, and $7^2 a^2 b^2 + c^3$ has degree 4 because 2 + 2 = 4 is greater than 3. The polynomial $x^B - Dx^A - C = x^2 - 4x - 5$, which factors into $(x-5)(x+1)$.

11) A. The discriminant is $b^2 - 4ac = 41^2 - 4(7)(333) = -8587$.
B. Use the discriminant again. It is 121, which is positive, so the equation has 2 solutions.
C. Again, use the discriminant. It is 1296, a perfect square, so there are 2 rational solutions.
D. This time, use the quadratic formula. The solutions are both $\frac{-3}{31}$, so that is the desired sum.
$AD - BC = (-8587)(\frac{-3}{31}) - (2)(2) = 827$.

12) Distribute the $\sqrt{2}$ to each entry of the second matrix. Add corresponding entries of the matrices and simplify: $\begin{pmatrix} \sqrt{50} & \sqrt{32} \\ 7\sqrt{3} & \sqrt{26} \end{pmatrix} + \sqrt{2}\begin{pmatrix} 3 & \sqrt{1} \\ \frac{12}{\sqrt{6}} & \sqrt{13} \end{pmatrix} = \begin{pmatrix} \sqrt{50} & \sqrt{32} \\ 7\sqrt{3} & \sqrt{26} \end{pmatrix} + \begin{pmatrix} \sqrt{6} & \sqrt{2} \\ 4\sqrt{3} & \sqrt{26} \end{pmatrix}$
$= \begin{pmatrix} \sqrt{50}+3\sqrt{2} & \sqrt{32}+\sqrt{2} \\ 7\sqrt{3}+4\sqrt{3} & \sqrt{26}+\sqrt{26} \end{pmatrix} = \begin{pmatrix} 8\sqrt{2} & 5\sqrt{2} \\ 11\sqrt{3} & 2\sqrt{26} \end{pmatrix}$. Now, $A-B+C-D+E-F+G-H=$
$8-2+5-2+11-3+2-26=-7$.

13) The order of operations is parentheses, exponents, multiplication/division, and addition/subtraction. Parenthesis has rank 1, exponents have rank 2, multiplication/division has rank 3, and addition/subtraction have rank 4. Now, the products are 25(1)=25, 4(2)=8, 16(3)=48, and 1(4)=4, and their sum is 85.

14) Use the order of operations to evaluate the expressions. A is -228. The expression in B has a value of 22, so its additive inverse is -22. The expression in C has a result of -4, so its multiplicative inverse is $\frac{-1}{4}$. The expression in D has a value of 5. Finally, $AC - BD = -228(\frac{-1}{4}) - (-22)(5) = 167$.

15) A. The slope of JL is $\frac{-2-1}{13-4} = \frac{-3}{9} = \frac{-1}{3}$. JL can now be written in point slope form using either point, and transforming it to standard form yields $x + 3y = 7$. $1 + 3 + 7 = 11$.
B. The slope of KM is $\frac{3-(-6)}{9-3} = \frac{9}{6} = \frac{3}{2}$. JK can also be written in point slope form; transforming it into standard form yields $3x - 2y = 21$. $3 + (-2) + 21 = 22$.
C. Use the equations found in the previous parts to make a system of equations. Solve the system; the point of intersection is (7, 0). $7 + 0 = 7$.
$(B/A)^C = (22/11)^7 = 2^7 = 128$.

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Algebra I Team
Answers

1) $x+3$
2) 549
3) -1
4) $65.1, 65\frac{1}{10}$, or $\frac{651}{10}$
5) 94
6) $\frac{15}{16}$
7) $\frac{330}{571}$
8) [150, 175]
9) 5
10) $(x-5)(x+1)$ or $(x+1)(x-5)$
11) 827
12) -7
13) 85
14) 167
15) 128