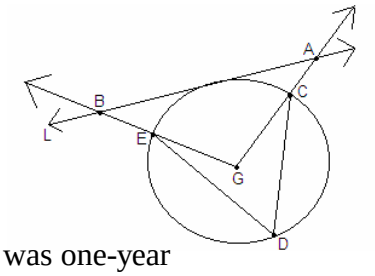
The abbreviation NOTA denotes "None Of These Answers."
Diagrams may not be drawn to scale.

1) In the figure below, line L is tangent to circle G, $\overline{AG} \cong \overline{GB}$, and m∠GBA=25°, then what is the degree measure of the supplement of ∠CDE ?



A. 115 B. 45 C. 55 D. 65 E. NOTA

2) A mother is now twice as old as her daughter. Ten years ago the mother was one-year less than three times the daughter's age. What is the sum of the mother and daughter's current age?

A. 42 B. 57 C. 60 D. 63 E. NOTA

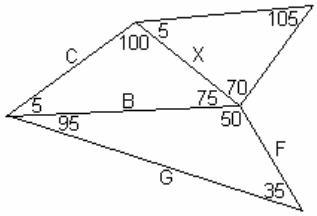
3) For $xy \neq 0,\ x \neq \pm 2y,\ y \neq \pm 2x,\ x \neq y$, $\dfrac{1+\dfrac{1}{1-\dfrac{x}{y}}}{1-\dfrac{3}{1-\dfrac{x}{y}}}$ =

A. $\dfrac{y+2x}{2x-y}$ B. $\dfrac{2x-y}{2x+y}$ C. $\dfrac{y-2x}{-x-2y}$ D. $\dfrac{x-2y}{x+2y}$ E. NOTA

4) Find $(i+1)^{10}$

A. 0 B. 1 C. $32i$ D. $-32i$ E. NOTA

5. Find the longest side of the figure shown.



A.G B. X C. B D. F E. NOTA

6) Find K so that the graph of $y = x^2 + \dfrac{K}{2}x + 2$ is tangent to the x-axis.

A. $\pm\sqrt{2}$ B. $\pm 2\sqrt{2}$ C. $\pm 4\sqrt{2}$ D. $\pm 4i\sqrt{2}$ E. NOTA

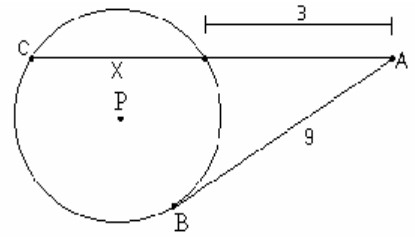
7) If *A*, *B* and *C* are distinct non-zero numbers which form a geometric progression in the order listed, find *C* in terms of *A* and *B*.

A. $\frac{B^2}{A}$ B. $\frac{A}{B^2}$ C. $\frac{A^2}{B}$ D. $\frac{2B}{A}$ E. NOTA

8) The sum of the coefficients in the expansion of $(x^2 - 2y)^7$ is ?

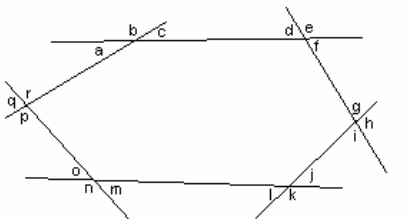
A. ${}_7C_3$ B. -3 C. -1 D. 0 E. NOTA

9) Solve for x. $\overline{AB}$ is a tangent and $\overline{AC}$ is a secant of circle P.



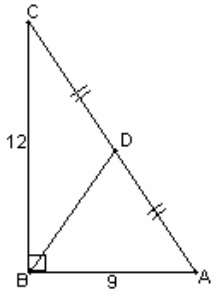
A. $\frac{-3+3\sqrt{37}}{2}$ B. 21 C. 24 D. 27 E. NOTA

10) In the planar figure shown below, $a + b + c + ... + p + q + r = x$.
Find *x*.



A. 720 B. 1440 C. 1080 D. 1260 E. NOTA

11) Find twice BD in the right triangle shown.



A. 18 B. 24 C. $\frac{15}{2}$ D. 15 E. NOTA

12) $\sqrt{6-\sqrt{6-\sqrt{6-\sqrt{6-\sqrt{...}}}}} =$

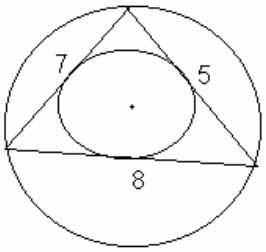
A. 4 B. 3 C. 6 D. 2 E. NOTA

13) Evaluate: $\sum_{K=1}^{11} (\sqrt{-1})^K$

A. -1 B. i C. $-i$ D. 1 E. NOTA

14) Let R be the radius of the inscribed circle and Let r be the radius of the circumscribed circle.

Find $R \div \frac{1}{r}$.



A. 14 B. 7 C. $\frac{6}{7}$ D. $\frac{3}{7}$ E. NOTA

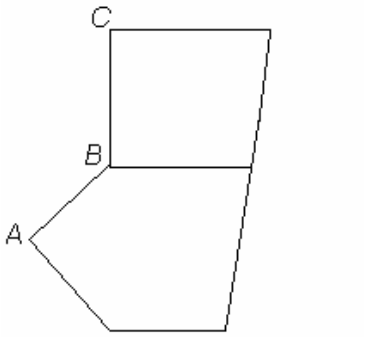
15) Simplify: $(1 + i)(3 - 3i)(2 - 2i^2)(-4 - 4i^2)$

A. 2 B. -2 C. 0 D. 4 E. NOTA

16) Find the area of the conic with equation $4x^2 + 8y^2 + 24x - 16y - 4 = 0$.

A. $9\pi\sqrt{2}$ B. 72π C. $6\pi\sqrt{2}$ D. 162π E. NOTA

17) Point B is a mutual vertex of a regular pentagon, a square, and a third regular polygon. If 2 of the sides of this polygon are $\overline{AB}$ and $\overline{BC}$, then the polygon has p sides.
Find p - 1.



A. 19 B. 20 C. 48 D. 49 E. NOTA

18) Find $\left|g^{-1}(0)\right|$, given that $f(x) = x^2 - 3$, $h(x) = 2x$ and $g(x) = f(h(x))$ for $x \geq 0$.

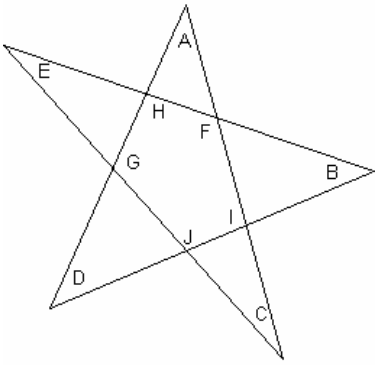
A. $\frac{1}{3}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{3}{2}$ D. 3 E. NOTA

19) Let p be the ratio of the radius to the circumference of circle E, and let g be the circumference of circle E. Let $x^2 + y^2 = 125$ be the equation of circle E.
Find pg.

A. $10\sqrt{5}$ B. $5\sqrt{5}$ C. $10\pi^2\sqrt{5}$ D. $\frac{\sqrt{5}}{25}$ E. NOTA

20) Let line P be defined by $y = 3x - 2$. Find the product of the slope and the y-intercept of the line perpendicular to P at the intersection point with abscissa of 1.

A. $\frac{-4}{9}$ B. -6 C. $\frac{-2}{3}$ D. $\frac{2}{3}$ E. NOTA



21) Find: $A - F + B - I + C - J + D - G + E - H$

A. -180 B. 50 C. -360 D. 90 E. NOTA

22) $a^2 * b = 2a - b^3$, for $a > 0$, and $d @ p = d^{\frac{1}{p-17}}$, then find the greatest absolute value for $9^3 @ (4 * -3)$.

A. 3 B. 9 C. 27 D. $9^{\frac{3}{14}}$ E. NOTA

23) Simplify. $125^{-\frac{2}{3}}$

A. -25 B. 25 C. $\frac{1}{25}$ D. $\frac{-1}{25}$ E. NOTA

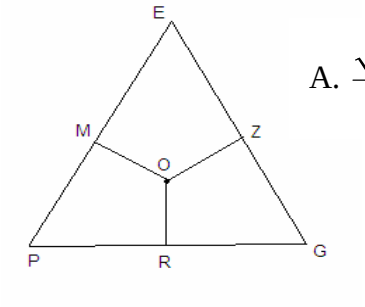
24. Evaluate $\sqrt{\frac{324 - x^2}{x^2 - 324}}$ for values of x such that $x^2 \neq 324$.

A. $\sqrt{(18 - x)(x - 18)}$ B. $\sqrt{\frac{4 + x}{x + 4}}$ C. i D. 1 E.NOTA

25) Find the coordinates of the vertex of the parabola with equation $y = 2x^2 + 3x - 1$.

A. $\left(-\frac{3}{2}, -\frac{19}{8}\right)$ B. $\left(-\frac{3}{4}, -\frac{17}{8}\right)$ C. $\left(-\frac{3}{2}, -\frac{19}{4}\right)$ D. $(0, -1)$ E. NOTA

26) Given that $\overline{OZ}, \overline{OR}$ and $\overline{OM}$ are perpendicular to $\overline{EG}, \overline{GP}$ and $\overline{PE}$, respectively. Find $m\overline{OZ} + m\overline{OR} + m\overline{OM}$, if ΔPEG is equilateral with side length 2.



A. $\frac{\sqrt{3}}{3}$ B. $2\sqrt{3}$ C. $\sqrt{3}$ D. $\frac{3}{2}$ E. NOTA

27) Find the product of the roots of $2x^3 - 7x^2 - 6x + 15 = 0$.

A. $\frac{7}{2}$ B. 3 C. $\frac{-15}{2}$ D. $\frac{15}{2}$ E. NOTA

28) Given: $F(x) = \sqrt{x-13}$ and $G(x) = 41x^7 + 13x^6 - 36x^5 + 54x^4 - 78x^3 + x^2 - 11x + 13x^0$, and $k =$ the 6th prime number. Find $G(F(G(F(G(F(k))))))$ to the nearest thousandth.

A. $2.086 \bullet 10^{51}$ B. 13 C. 0 D. $9.312 \bullet 10^{39} - 7.053 \bullet 10^{39} i$ E. NOTA

29) Find the sum of the first 40 natural numbers.

A. 820 B. 780 C. 1640 D. 1560 E. NOTA

30) Find the length of the major arc formed by the hands of a clock with a diameter of 12 units and the time is 1:47 pm.

A. $\frac{491\pi}{60}$ B. $\frac{577\pi}{60}$ C. $\frac{457\pi}{60}$ D. $\frac{263\pi}{60}$ E. NOTA

Solutions

1. A

$\overset{\frown}{CE}$ = 130 degrees by definition of central angle, since $\triangle AGB$ is an isosceles triangle.
$\angle CDE = 65$ by definition of inscribed angle. So the supplement is 180-65=115.

2. D

$M = 2D \quad M - 10 = 3(D - 10) - 1$
$2D - 10 = 3D - 31$
$D = 21, M = 42. \quad D + M = 21 + 42 = 63$

3. D

$$\frac{1+\dfrac{1}{1-\dfrac{x}{y}}}{1-\dfrac{3}{1-\dfrac{x}{y}}} \rightarrow \frac{\dfrac{y-x}{y-x}+\dfrac{y}{y-x}}{\dfrac{y-x}{y-x}-\dfrac{3y}{y-x}} \rightarrow \frac{\dfrac{2y-x}{y-x}}{\dfrac{-2y-x}{y-x}} \frac{2y-x}{-2y-x}$$

Which is equivalent to $\dfrac{x-2y}{x+2y}$.

4. C

$(i+1)^{10} \rightarrow \left((i+1)^2\right)^5 \rightarrow (2i)^5 = 32i$

5. D

x is the largest side of the 5-70-105; however, the smallest side of the 5-75-100. B is the largest side of the 5-75-100; however, the smallest one of the 35-50-95. So the largest side is F, since it is across from the largest angle (95).

6. C

$\left(\dfrac{K}{4}\right)^2 = 2 \rightarrow K^2 = 16 \bullet 2 \rightarrow K = \pm 4\sqrt{2}$

7. A

The ratio in a geometric progression is the quotient of a_{n+1} and a_n.
So $r \bullet B \rightarrow \dfrac{B}{A} \bullet B = \dfrac{B^2}{A}$

8. C

$\left(x^2 - 2y\right)^7 \rightarrow (1-2)^7 \rightarrow (-1)^7 \rightarrow -1$

9. C

$9^2 = 3(3+x) \rightarrow 81 = 9 + 3x \rightarrow 72 = 3x \rightarrow x = 24$

10. B

The sum of every three-pair angle and the unmarked angle is equal to 360.
So the sum of all the letters is equal to $360(6) - 180(6-2) = 1440$

11. D

By the definition of a median in a right-triangle, its measure is half of the length of the hypotenuse. The 9-12-15 triangle has a median of $\frac{15}{2}$. Twice $\frac{15}{2}$ is 15.

12. D

$\sqrt{6-\sqrt{6-\sqrt{6-\sqrt{6-\sqrt{...}}}}} = x \rightarrow 6 - x = x^2$

$x^2 + x - 6 = 0 \rightarrow (x+3)(x-2) = 0.\ x = 2,$ becasue a $\sqrt{...}$ cannot be negative.

13. A

$\sum_{k=1}^{11} i^k$ every 8 terms the sum goes to zero. The pattern is i, -1, -i, 1.

So the sum of the first eleven terms would be -1.

14. B

(This is a MEAN one!!) R = inscribed, r = Circumscribed.

$R = \frac{2A}{a+b+c}$ (where a, b and c are the sides and A is the area of the triangle)

r = $\frac{abc}{4A}$ R $\div \frac{1}{\text{r}} = Rr = \frac{2A}{a+b+c} \cdot \frac{abc}{4A} = \frac{abc}{2(a+b+c)}$

$Rr = \frac{5 \cdot 7 \cdot 8}{2(5+7+8)} = 7$

15. C

$(1 + i)\ (3 - 3i)\ (2 - 2i^2)\ (-4 - 4i^2)$

$(-4 - 4i^2) \rightarrow (-4 + 4) \rightarrow 0$. The product is zero.

16. C

$4x^2 + 8y^2 + 24x - 16y - 4 = 0 \rightarrow \frac{(x+3)^2}{12} + \frac{(y-1)^2}{6} = 1$

The area of the ellipse is $ab\pi \rightarrow \sqrt{12} \bullet \sqrt{6} \bullet \pi \rightarrow 6\pi\sqrt{2}$

17. A

$360 - (90 + 108) = 162$

$\frac{180(n-2)}{n} = 162 \rightarrow -360 = -18n \rightarrow n = 20.$ $n - 1 = 19$

18. B

$g(x) = f(h(x)) = 4x^2 - 3 \rightarrow x = 4y^2 - 3 \rightarrow \frac{x+3}{4} = y^2$

$g^{-1}(x) = \pm\frac{\sqrt{x+3}}{2} \rightarrow \left|g^{-1}(0)\right| = \frac{\sqrt{3}}{2}$

19. B

$p = \frac{r}{2\pi r} \rightarrow \frac{1}{2\pi}$ $g = 2\pi r$

$pg = \frac{2\pi r}{2\pi} \rightarrow r = \sqrt{125} = 5\sqrt{5}$

20. A

$y = 3x - 2,$ when x=1, y=1.

$y - y_1 = \frac{-1}{3}(x - x_1) \rightarrow y = \frac{-1}{3}x + \frac{4}{3}$. The product of the slope and the y-intercept is $\frac{-1}{3} \bullet \frac{4}{3} = \frac{-4}{9}$

21. C

$(A + B + C + D + E) - (F + G + H + I + J) \rightarrow (180) - (540) = -360$

$729 @ (4 * -3),\ ((\pm 2)^2 * (-3)) = 2(\pm 2) - (-27) \rightarrow$ *so* 31 *or* 23.

22. A

$729@23 = 729^{\frac{1}{6}} = (3^6)^{\frac{1}{6}} = 3 \qquad 729 @ 31 = 729^{\frac{1}{14}} = (3^6)^{\frac{1}{14}} = 3^{\frac{3}{7}} \qquad 3^1 > 3^{\frac{3}{7}}$

$\therefore$ 3 gives a greater absolute value.

23. C

$125 = 5^3 \rightarrow (5)^{3 \bullet \frac{-2}{3}} = 5^{-2} = \frac{1}{25}$

24. C

$\sqrt{\frac{-1(x^2 - 324)}{(x^2 - 324)}} = i$

25. B

$2x^2 + 3x - 1$ has a vertex at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \rightarrow \left(\frac{-3}{4}, \frac{-17}{8}\right)$

26. C

$m\overline{OZ} + m\overline{OR} + m\overline{OM} =$ The altitude of the equilateral triangle. so $\sqrt{3}$

27. C

The product of the roots is $\frac{-z}{a}$, where z is the constant and a is the leading coeffiecient.

$2x^3 - 7x^2 - 6x + 15 \rightarrow \frac{-15}{2}$

28. B

$F(x) = \sqrt{x - 13}$ *and* $G(x) = 41x^7 + 13x^6 - 36x^5 + 54x^4 - 78x^3 + x^2 - 11x + 13x^0,\ k = 13$

$G(F(G(F(G(F(13)))))) = G(F(G(F(G(0))))) = G(F(G(F(13)))) = G(F(G(0)))$

$G(F(13)) = G(0) = 13$

29. A

$\sum_{p=1}^{40} p = \frac{n(a_1 + a_n)}{2} \rightarrow \frac{40(41)}{2} = 820$

$7(30) + 12 + \frac{(60 - 47)}{60}(30) \rightarrow 222 + \frac{13}{2} = \frac{457}{2}$

30. C

$\frac{457}{2 \bullet 360} \bullet 12\pi = \frac{457\pi}{60}$

Answer to
Theta Open

1. A
2. D
3. D
4. C
5. D
6. C
7. A
8. C
9. C
10. B
11. D
12. D
13. A
14. B
15. C
16. C
17. A
18. B
19. B
20. A
21. C
22. A
23. C
24. C
25. B
26. C
27. C
28. B
29. A
30. C