

Geometry Individual Test NO CALCULATOR!!!
 Middleton Invitational February 18, 2006

The abbreviation "NOTA"
 denotes "None of These
 Answers."

Diagrams are not necessarily
 drawn to scale.

1. Which set does **not** always define a plane?

- A. three noncollinear points.
- B. two distinct intersecting lines.
- C. two parallel lines
- D. a line and a point
- E. NOTA

2. Parallelogram ABCD has $m\angle A = 70^\circ$. If $m\angle B = (2x + 10)^\circ$ then $x =$

- A. 30 B. 40
- C. 50 D. 70 E. NOTA

3. An equilateral triangle has perimeter 9. What is the height of the triangle?

- A. 4.5 B. $3\sqrt{3}$
- C. $\frac{9\sqrt{3}}{2}$ D. $\frac{3\sqrt{3}}{2}$ E. NOTA

4. A quadrilateral RSTU is inscribed in a circle. If $m\angle R = 30^\circ$ and $m\angle S = 40^\circ$ then give the $m\angle T$.

- A. 30° B. 40°
- C. 140° D. 150° E. NOTA

5. In right $\triangle ABC$, \overline{AB} is the hypotenuse and is 2 cm more than the longest leg. The other leg is half of \overline{AB} . Give the length of the shortest leg, in cm.

- A. $4\sqrt{2} + 3$ B. $4\sqrt{3}$
- C. $2\sqrt{3} + 4$ D. $\sqrt{3} + 6$ E. NOTA

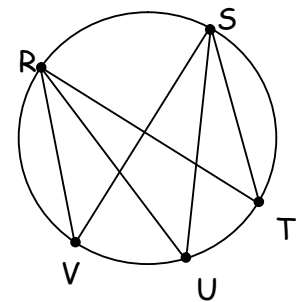
6. The area of a regular rhombus is half of the area of a triangle. One side of the rhombus is 15. If one side of that triangle has length 12, then find the height to that side.

- A. 78 B. 75
- C. 74 D. 72 E. NOTA

7. A regular pentagon has one interior angle which has measure $2x$ degrees. Give the value of $x + 8$.

- A. 62 B. 61
- C. 60 D. 59 E. NOTA

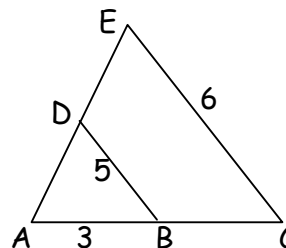
8. In the circle shown, $m\angle RVS = a^\circ$.



Find an expression for $m\angle RVS + m\angle RUS - m\angle RTS$.

- A. a° B. $2a^\circ$
- C. $3a^\circ$ D. 0 E. NOTA

9.



\overline{DB} is parallel to \overline{EC} and $AB=3$, $DB=5$, and $EC=6$. Find the length BC to the nearest tenth place.

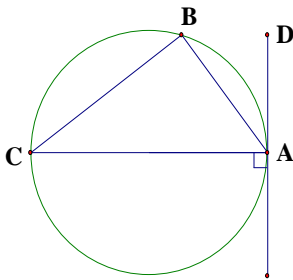
- A. 4.0 B. 3.6
- C. 1.7 D. 0.6 E. NOTA

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10. Given rectangle ABCD with AB = 15 cm and BC = 8 cm, circles with centers at E and F are inscribed in $\triangle ABD$ and $\triangle BCD$. Find EF in cm.

- A. $\sqrt{85}$ cm B. $6\sqrt{2}$ cm
C. $\sqrt{82}$ cm D. $5\sqrt{3}$ cm E. NOTA

11. \overline{DA} is tangent to the circle at A. The measure of arc BC is 112° . Find $m\angle BAD$.



- A. 34° B. 45°
C. 56° D. 68° E. NOTA

12. A circle passes through two vertices and the orthocenter of an equilateral triangle with sides of length $2\sqrt{3}$ cm. Find the area of this circle.

- A. 4π cm² B. $2\sqrt{3}\pi$ cm²
C. $3\sqrt{2}\pi$ cm² D. 3π cm²
E. NOTA

13. An ornithologist stands 280 ft from the face of a vertical cliff and observes an eagle's nest on a ledge. She measures the tangent of the angle of elevation to be $3/7$. If the height of her eye is 5 ft when she measures the tangent, how high is the nest?

- A. 120 ft B. 125 ft
C. 137 ft D. 140 ft E. NOTA

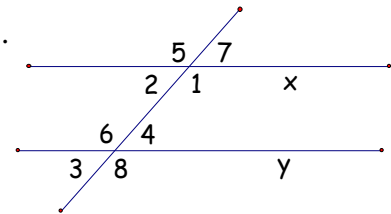
14. The incenter of $\triangle ABC$ is located at point D. If $m\angle C = 32^\circ$, find $m\angle ADB$.

- A. 106° B. 116°
C. 148° D. 164° E. NOTA

15. Triangle ABC contains sides AB = 8 cm, BC = 12 cm, AC = 16 cm. M is the midpoint of \overline{AC} . Find the length of \overline{MB} .

- A. $2\sqrt{10}$ cm B. 8 cm
C. $4\sqrt{3}$ cm D. 6 cm E. NOTA

16. If $x \parallel y$, $m\angle 1 = 111 - a$ and $m\angle 4 = 4a - 42$, then find $m\angle 5$.

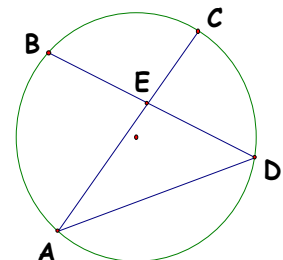


- A. 37° B. 74°
C. 106° D. 143° E. NOTA

17. A right rectangular prism with dimensions (in cm) of 12, 15 and 16 is inscribed in a sphere. Find the surface area of the sphere.

- A. 625π cm² B. 440π cm²
C. $360\sqrt{3}\pi$ cm² D. $32\sqrt{185}\pi$ cm²
E. NOTA

18. In the diagram $m\widehat{AB} = 108^\circ$ and $m\widehat{CD} = 64^\circ$. Find $m\angle AEB$.

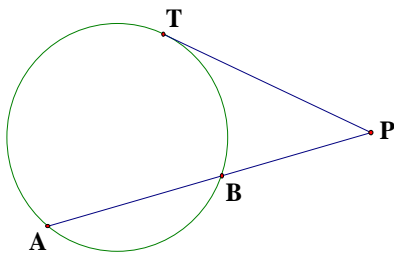


- A. 84° B. 86°
C. 94° D. 106° E. NOTA

19. An altitude of a right triangle intersects the hypotenuse at a point that divides it into a ratio of 4 to 9. If the hypotenuse measures 7, find the length of the altitude.

- A. $\frac{42}{13}$ B. $\frac{63}{4}$
 C. $\frac{28}{9}$ D. $\frac{36}{7}$
 E. NOTA

20. Secant \overline{PA} intersects a circle at points A and B so that AB is 19 cm and PB is 8 cm. Find the length of tangent segment \overline{PT} .



- A. 13.5 cm B. $9\sqrt{3}$ cm
 C. 11 cm D. $6\sqrt{6}$ cm
 E. NOTA

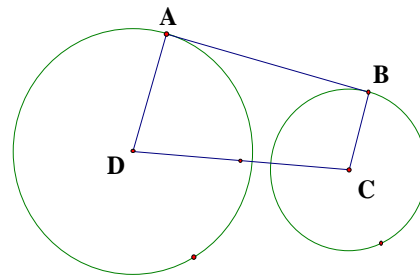
21. A regular octagon has a side of length 12 cm. Find its area.

- A. $(288 + 288\sqrt{2}) \text{ cm}^2$
 B. $(144 + 576\sqrt{2}) \text{ cm}^2$
 C. $(288 + 144\sqrt{2}) \text{ cm}^2$
 D. $(576 + 144\sqrt{2}) \text{ cm}^2$
 E. NOTA

22. ABCD is a cyclic quadrilateral whose diagonals intersect at P. If AP is 6, BP is 8 and AC is 15, find length DP.

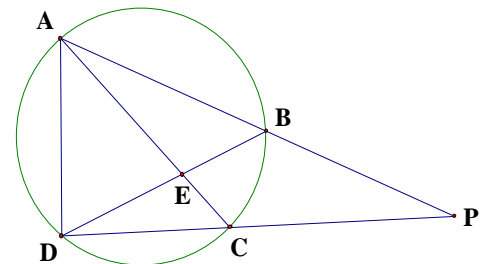
- A. 7 B. $6\frac{3}{4}$
 C. $\frac{15}{8}$ D. $11\frac{1}{4}$
 E. NOTA

23. \overline{AB} is externally tangent to circles C and D as shown in the figure below. If the radius of C is 8 cm, the radius of D is 15 cm and the distance between the centers is 25 cm, find the area of quadrilateral ABCD.



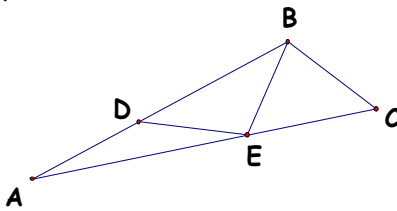
- A. 287.5 cm^2 B. 552 cm^2
 C. 575 cm^2 D. 276 cm^2
 E. NOTA

24. In the diagram below, A, B, C, and D are on the circle, $m\angle ABD = 70^\circ$ and $m\angle BEA = 80^\circ$. Find $m\angle P$.



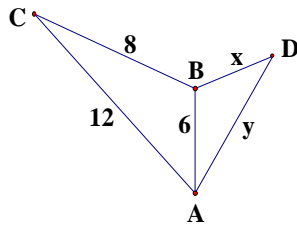
- A. 40° B. 25°
 C. 75° D. 30°
 E. NOTA

25. In triangle ABC, $AD = DE = EB = BC$. If the measure of $\angle ABC$ is 100° , what is the measure of $\angle BAC$?



- A. 15° B. 20°
 C. 25° D. 30°
 E. NOTA

26. In the figure below, $\triangle ABC \sim \triangle DBA$. Find the sum of x and y .



- A. 13.5 B. 13
 C. $9\frac{1}{2}$ D. 10
 E. NOTA

27. In right $\triangle ABC$, $\sin A = \cos B = \frac{\sqrt{3}}{2}$.

If $AC = 8$, find BC .

- A. $\frac{9}{4}$ B. $4\sqrt{3}$
 C. $8\sqrt{3}$ D. 4
 E. NOTA

28. The quadrilateral formed by connecting the midpoints of the consecutive sides of a concave quadrilateral, none of which are congruent, is always a:

- A. rectangle B. rhombus
 C. parallelogram D. trapezoid
 E. NOTA

29. Floyd hates English class, so he began to watch the (analog) clock at 11:50 am. What is the fewest number of minutes that will elapse before the hands of the clock are at right angles?

- A. $26\frac{2}{9}$ B. $26\frac{4}{11}$
 C. $25\frac{3}{5}$ D. $24\frac{6}{13}$
 E. NOTA

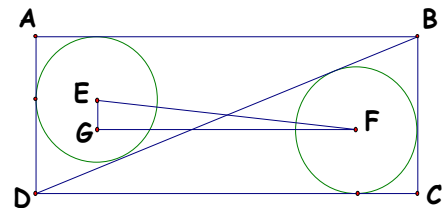
30. A plane contains a number of straight lines, no two of which are parallel and no three of which are concurrent. If these lines intersect in 153 distinct points, how many lines are in the plane?

- A. 18 B. 17
 C. 16 D. 15
 E. NOTA

Solutions

1. D. The point could be on the line.
2. C. Angles A and B are supplementary. $2x + 10 = 180 - 70$. $x = 50$
3. D. Perimeter 9 means side is 3. Altitude divides the triangle into a 30-60-90 with short leg $3/2$. Long leg is $(3/2)(\sqrt{3})$
4. D. Opposite angles of an inscribed quadrilateral are supplementary. $m\angle T = 180 - 30$
5. C. Let $x =$ short leg. The hypotenuse is $2x$ (the "other" leg is half the hypotenuse \overline{AB}) and the long leg is $2x - 2$ (2 less than the hypotenuse). Pythagorean Theorem $x^2 + (2x - 2)^2 = (2x)^2$ simplifies to the quadratic $x^2 - 8x + 4 = 0$. Quadratic Formula yields $x = 4 \pm 2\sqrt{3}$. While $4 - 2\sqrt{3}$ is also a solution, it is not a choice. $2\sqrt{3} + 4$ is a choice.
6. B. A regular rhombus is a square. Side length is 15. Area 225. The area of the triangle is 450 (twice the area of the square.) Using the triangle side of 12 as the base, h as the length of the altitude to that side, and the formula $A = (1/2)bh$ leads to $(1/2)(12)h = 450$. $h = 75$.
7. A. One interior angle of a regular pentagon is 108. $2x = 108$. $x = 54$. $x + 8 = 62$.
8. A. Since $\angle RVS$ is an inscribed angle, the arc it intercepts is $2a$. $\angle RUS$ and $\angle RTS$ intercept the same arc. They also measure a . $a + a - a = a$
9. D. $\triangle ABD \sim \triangle ACE$. Let $x = m\overline{BC}$. $\frac{3}{3+x} = \frac{5}{6}$. $5x + 15 = 18$. $x = 0.6$

10. A. Diagonal \overline{DB} divides the rectangle into congruent right triangles with sides of 15 and 8. \overline{DB} is the hypotenuse of the Pythagorean triple. Solving, $m\overline{DB}$ is 17. The radius of a circle inscribed in a right triangle is half of (the sum of the legs minus the hypotenuse). $r = (1/2)(15 + 8 - 17) = 3$



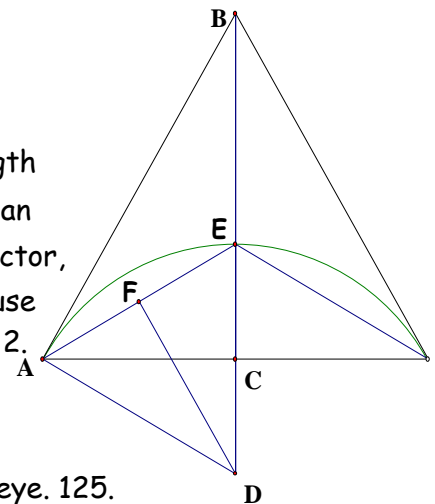
\overline{EF} is the hypotenuse of right triangle EGF. See diagram.

$m\overline{EG} = 8 - 2r = 8 - 6 = 2$. $m\overline{GF} = 15 - 2r = 15 - 6 = 9$.

By Pythagorean Theorem, $m\overline{EF} = \sqrt{2^2 + 9^2} = \sqrt{85}$

11. A. \overline{CA} is perpendicular at the point of tangency. It must pass through the center making it a diameter and $\widehat{ABC} = 180^\circ$. Subtracting $m\widehat{BC}$, $180^\circ - 112^\circ = 68^\circ = m\widehat{AB}$. $m\angle BAD$ is half of the arc that it intercepts, making it 34° .

12. A. See diagram. $m\widehat{AB}$ is given as $2\sqrt{3}$, so $m\widehat{AC} = \sqrt{3}$ and the length of the altitude \overline{BC} is 3. Since the orthocenter is also the centroid of an equilateral triangle, \overline{BE} is 2 and \overline{AE} is also 2. The perpendicular bisector, \overline{FD} , of chord \overline{AE} passes through the center of the circle at D. Because $\angle FED$ is 60° , $\triangle FED$ is a 30-60-90 and $\triangle AED$ is equilateral with sides = 2. This is also the radius of the circle. $A = \pi r^2 = 4\pi$.



13. B. Tangent is opposite (height of the nest) over adjacent (distance to the cliff). Using a proportion, $\frac{3}{7} = \frac{h}{280}$. $h = 120$ plus the height of eye. 125.

14. A. The incenter is formed using the angle bisectors. Let the angle at A measure $2a$ and the angle at B measure $2b$. Since C measures 32° , $2a + 2b = 180^\circ - 32^\circ$. $a + b = 74^\circ$. Looking at the triangle formed by the vertices A, D and B, $m\angle ADB + a + b = 180^\circ$. $m\angle ADB = 106^\circ$.

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15. A. \overline{MB} is a median. By formula, the length of a median is $\sqrt{\frac{a^2}{2} + \frac{b^2}{2} - \frac{c^2}{4}}$ where a and b are the sides that form the vertex from which the median is connected to the midpoint of the opposite side, c.

$$m\overline{MB} = \sqrt{\frac{8^2}{2} + \frac{12^2}{2} - \frac{16^2}{4}} = 2\sqrt{10}.$$

16. B. $\angle 1$ and $\angle 4$ are supplementary. $111 - a + 4a - 42 = 180$. $a = 37$. $\angle 5$ is vertical with $\angle 1$, so they are congruent. $m\angle 5 = 111 - 37 = 74$.

17. A. The space diagonal, d, of the prism is the diameter of the sphere. $d = \sqrt{12^2 + 15^2 + 16^2} = 25$. The radius is $25/2$. Surface area of a sphere is $4\pi r^2 = 625\pi$.

18. B. $\angle CAD$ and $\angle BDA$ are inscribed and equal to half the arcs they intercept. $m\angle CAD = 64/2 = 32$. $m\angle BDA = 108/2 = 54$. These two angles are the remote interior angles for exterior $\angle AEB = 32 + 54 = 86$

19. A. Let $x =$ short part of the hypotenuse. Then, $\frac{x}{7-x} = \frac{4}{9}$. $x = \frac{28}{13}$ and $7-x = \frac{63}{13}$. Since the

height, h, is equal to the square root of the product of x and $x - 7$, $h = \sqrt{\frac{9 \cdot 7}{13} \cdot \frac{7 \cdot 4}{13}} = \frac{3 \cdot 7 \cdot 2}{13} = \frac{42}{13}$

20. D. $(m\overline{PT})^2 = (m\overline{PB})(m\overline{PA}) = 8(8 + 19)$.

$$m\overline{PT} = \sqrt{8 \cdot 27} = \sqrt{4 \cdot 2 \cdot 3 \cdot 9} = 6\sqrt{6}$$

21. A. See diagram. Constructing a square around the octagon, the square has sides of length $12 + 12\sqrt{2}$. The area of the octagon is the area of the square minus the four triangles at the corners. $(12 + 12\sqrt{2})^2 - 4[\frac{1}{2}(6\sqrt{2})^2]$

$$\text{Area} = 144 + 288\sqrt{2} + 288 - 144 = 288 + 288\sqrt{2}$$

22. B. The intersection of the diagonals divides each diagonal into two parts. The product of these parts is always equal.

$$AP(CP) = BP(DP). DP = 6(15-6)/8 = 27/4 = 6\frac{3}{4}.$$

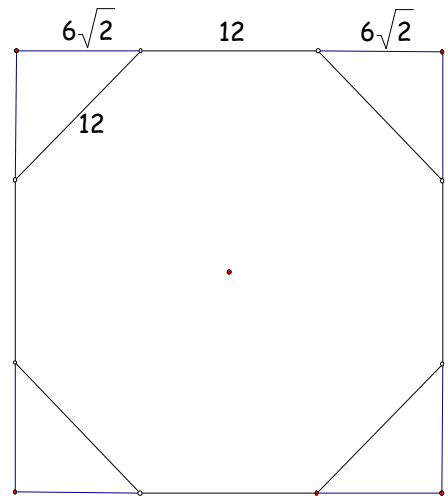
23. D. \overline{AD} and \overline{BC} are both perpendicular to the same segment, so they are parallel and ABCD is a trapezoid. Put a point E on \overline{AD} so that \overline{CE} is parallel to \overline{AB} . ABCE is a rectangle. $DE = 15 - 8 = 7$. The right triangle DEC has a hypotenuse of 25 and one leg of 7. By the Pythagorean Theorem, the other leg is 24 which is the height of the trapezoid. Area of the trapezoid is $\frac{1}{2}(8 + 15)(24) = 276$.

24. A. $m\angle BAE = 180 - 70 - 80 = 30$. $\angle BDC$ intercepts the same arc, so it is also 30. $\angle DBP$ is supplementary with the 70, so it is 110. $m\angle P = 180 - 110 - 30 = 40$.

25. B. Let $m\angle A = a$ and $m\angle C = c$. Then $a + c + 100^\circ = 180^\circ$. $a + c = 80^\circ$. $m\angle AED = a$ and $m\angle BEC = c$ because $\triangle AED$ and $\triangle BEC$ are isosceles. $m\angle DEB = 180 - (a + c) = 100^\circ$. $m\angle DBE = m\angle BDE = (180^\circ - 100^\circ)/2 = 40^\circ$. $m\angle BDE = 2a$ because it is an exterior angle with both remote interior angles = a. $2a = 40^\circ$. $a = 20$.

26. A. Corresponding sides of similar triangles are proportional. $\frac{AB}{DB} = \frac{BC}{BA} = \frac{AC}{DA}$ and $\frac{6}{x} = \frac{8}{6} = \frac{12}{y}$

$$x = 36/8 = 4.5 \text{ and } y = 72/8 = 9. \text{ } x + y = 13.5$$



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27. C. The right angle is at C. $\triangle ABC$ is similar to a triangle with hypotenuse 1 and long leg $\sqrt{3}/2$. The short leg is $1/2$. $\triangle ABC$ is a 30-60-90 with the 60 at A. Short leg is 8, Long leg is $8\sqrt{3}$.
28. C. Consider concave quadrilateral ABCD (a dart) with \overline{DB} the diagonal that is contained within the quadrilateral. (Diagonal \overline{AC} is outside.) The segment from the midpoint of \overline{AB} to the midpoint of \overline{AD} is a midsegment of $\triangle ABD$, parallel to \overline{DB} and half its length. The segment from the midpoint of \overline{CB} to the midpoint of \overline{CD} is a midsegment of $\triangle CBD$, parallel to \overline{DB} and half its length. Segments parallel to the same segment are parallel to each other. Both are half of DB thus congruent. Segments of a quadrilateral that are both parallel and congruent define a parallelogram.
29. B. The minute hand starts out $5/6$ of the way around the clock face, thus it is pointing at the 10. The hour hand begins $5/6$ of the way from the 11 to the 12. Considering the face to be 360, the minute hand begins at 300 and the hour hand at 355. The 55 initial angle is decreasing as time continues. To reach a point where the hands are perpendicular, the minute hand must pass the hour hand and continue increasing the angle until the degree measure of the minute hand minus the degree measure of the hour hand is 90. Solve. $(300 + 6m) - (355 + .5m) = 90$. $5.5m = 145$. $m = 1450/55 = 26\frac{4}{11}$.
30. A. One line = 0 intersections; 2 lines = 1 intersection; 3 lines = 3 intersections. Triangular numbers. Double and factor to get $n(n-1)/2 = 153$. Solve. $n(n-1) = 306$. Factors of 306 which are different by 1 are 17 and 18. The larger factor is n. $n = 18$.

1. Let the points of concurrency for a triangle be represented by these letters:
A for incenter, B for orthocenter, C for centroid and D for circumcenter. Record these letters in order to match the descriptions.

First: intersection of the medians

Second: intersection of the angle bisectors

Third: intersection of the perpendicular bisectors of the sides

Fourth: intersection of the altitudes

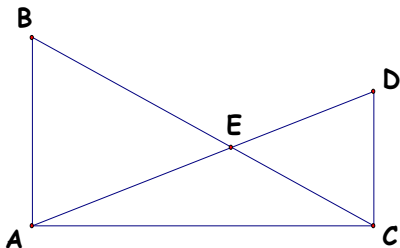
2. A = The number of vertices in a polyhedron with 12 faces and 24 edges
B = The number of diagonals in a dodecagon
C = The measure of one exterior angle of a regular nonagon
D = The sum of the first six triangular numbers

Find $A + B + C + D$

3. A = the height in cm of a cylinder with a diameter of 12 cm and a volume of $180\pi \text{ cm}^3$
B = the radius in cm of the base of a cone with a slant height of 9 cm and a surface area of $90\pi \text{ cm}^2$
C = the side, in cm, of the square base of a pyramid with a height of 12 cm and a volume of 400 cm^3 .
D = the width in cm of a right rectangular prism with a length of 12 cm, height of 7 cm and surface area of 472 cm^2 .

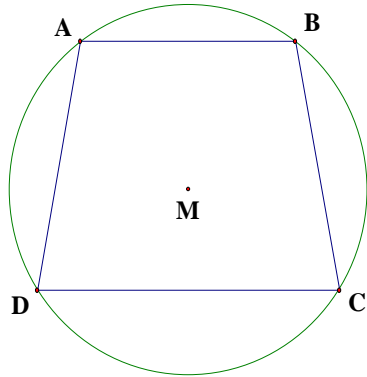
Find $A + B + C + D$

4. Angle BAC and angle DCA are right angles. If $AB = 10$, $AC = 12$ and $DC = 6$, find the area of triangle ACE.



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5. Circle M is circumscribed about trapezoid ABCD with AB = 10 and CD = 22 and a height of 12. Find the area of the circle.



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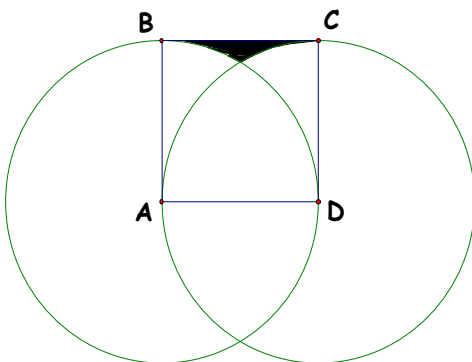
6. The equations of two circles are given below. Let the point (A, B) represent the center of the first circle and C equal the radius. For the second circle, let the point (D, E) represent the center and F equal the radius. Find $A + B + C + D + E + F$.

$$y^2 - x + 13 = 4y - x^2 + 9x$$

$$x^2 + 6y - 14 = 8x - y^2 - 3$$

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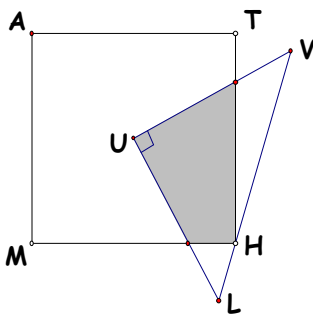
7. Circles A and D of radius 10 cm intersect in such a way that their centers are 10 cm apart. What is the area of the shaded region inside of ABCD and outside of both circles?



8. Consider the following six statements. If the statement is true, add the value in parentheses. If it is false, subtract the value. Find the total.
- A. Diagonals of a parallelogram bisect each other (5)
 - B. Consecutive angles of a kite are supplementary (13)
 - C. Opposite angles of a rhombus are congruent (23)
 - D. Diagonals of a rectangle are perpendicular (37)
 - E. An isosceles trapezoid has exactly one line of symmetry (41)
 - F. The exterior angle of a regular dodecagon measures 30° (59)

9. Find the area of a triangle formed by the x-axis, y-axis and the line $y = -\frac{3}{2}x + 9$.

10. The 90° vertex of isosceles right triangle LUV is the center of square MATH. If the square has a side of length 10 cm and point H is on the hypotenuse of the right triangle, what is the area of the overlapping (shaded) region?

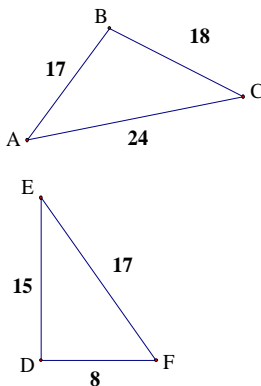


11. An industrious (and very worried) squirrel begins storing nuts for the winter. On day 1 he stores 5 nuts. The next day he realizes that he must pick up the pace to have enough nuts to last the winter. He stores 13 nuts on day 2, 17 on day 3 and 21 on day 4. If he continues in this pattern, how many TOTAL will he have stored after 30 days?

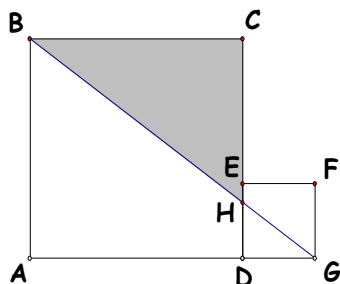
12. A kite has consecutive sides which measure 20 and 30 cm. If one vertex angle measures 90° , Find the area of the kite.

13. Find the ordered pair for the centroid of a triangle whose vertices are located at $(-13, 7)$, $(22, -14)$ and $(-15, 13)$

14. $\triangle ABC \sim \triangle MNP$ ($\triangle MNP$ which is not shown) If one side of $\triangle MNP$ is 51 cm, and all sides are integers, then K is the length of the largest possible side of $\triangle MNP$. $\triangle DEF \sim \triangle XYZ$ (also, not shown) If $YZ = 85$ cm, then H is the largest possible altitude of $\triangle XYZ$. Find $K + H$.



15. The sides of squares ABCD and DEFG are 3 cm and 1 cm respectively. Find the area of triangle BCH expressed as a fraction.



Solutions: Middleton Invitational 2-18-2006 Geometry Team SOLUTIONS

1. C A D B from definitions

2. Euler's Formula for solids: vertices + faces = edges + 2

$$A \text{ (number of vertices)} + 12 = 24 + 2 \quad A = 14$$

$$\text{For diagonals in an n-gon, use } n(n-3)/2 \quad \text{For a dodecagon, } n = 12. \quad B = 12(12-3)/2 = 54$$

$$C = 360/9 = 40 \quad D = 1 + 3 + 6 + 10 + 15 + 21 = 56$$

$$A + B + C + D = 14 + 54 + 40 + 56 = 164$$

3. Volume of a cylinder = $\pi r^2 h$, where h is the height. $180\pi = \pi \left(\frac{12}{2}\right)^2 h$. $A = 5$

Surface area of a cone = $\pi r^2 + \pi r l$ where r = radius of the base of the cone and l is the slant height. $\pi r^2 + \pi r(9) = 90\pi$ Divide pi out of both sides and solve the quadratic.

$$r^2 + 9r - 90 = 0. \text{ Factors are } (r + 15) \text{ and } (r - 6). \text{ For a positive } r, r = 6. \quad B = 6$$

Volume of a square pyramid = $\frac{1}{3}s^2 h$ where s = side length of the square base and h is the height. $400 = \frac{1}{3}s^2(12)$. $100 = s^2 \quad s = 10 \quad C = 10$

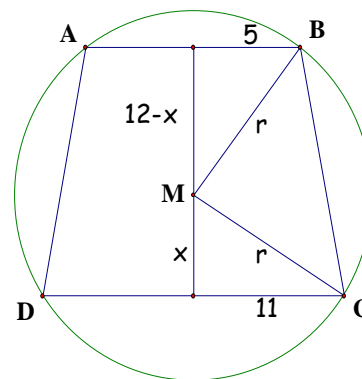
$$\text{Surface area} = 2(lw) + 2(lh) + 2(wh). \quad 472 = 2(12)w + 2(12)(7) + 2(w)(7). \quad w = 8 \quad D = 8$$

$$A + B + C + D = 5 + 6 + 10 + 8 = 29$$

4. First find the height of triangle ACE. $h = \frac{10 \cdot 6}{10 + 6}$ The area of the triangle is

$$\frac{1}{2} \cdot 12 \cdot h = 6 \cdot \frac{15}{4} = \frac{45}{2} \text{ or } 27 \frac{1}{2}$$

5. To find the radius begin by constructing the height through the center of the circle and solving two right triangle problems to find x. $r^2 = 5^2 + (12-x)^2$ and $r^2 = 11^2 + x^2$ so that $25 + 144 - 24x + x^2 = 121 + x^2$ and $x = 2$. $r^2 = 121 + 4 = 125$ Area = $\pi r^2 = 125\pi$



6. Combine like terms. Arrange in circle format. Complete the square.

$$x^2 - 10x + \underline{\quad} + y^2 - 4y + \underline{\quad} = -13$$

$$x^2 - 8x + \underline{\quad} + y^2 + 6y + \underline{\quad} = -3 + 14$$

$$x^2 - 10x + \underline{25} + y^2 - 4y + \underline{4} = -13 + 25 + 4$$

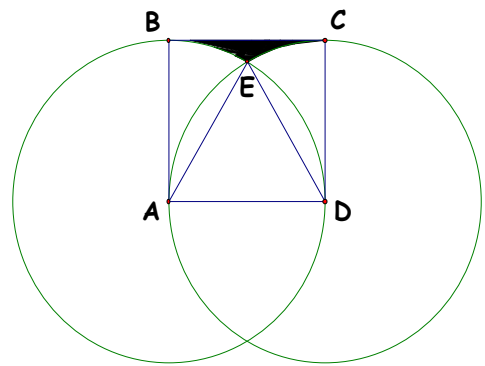
$$x^2 - 8x + \underline{16} + y^2 + 6y + \underline{9} = 11 + 16 + 9$$

$$(x - 5)^2 + (y - 2)^2 = 16 \text{ center } (5, 2) \text{ radius } 4$$

$$(x - 4)^2 + (y + 3)^2 = 36 \text{ center } (4, -3) \text{ radius } 6$$

$$A + B + C + D + E + F = 5 + 2 + 4 + 4 + (-3) + 6 = 18$$

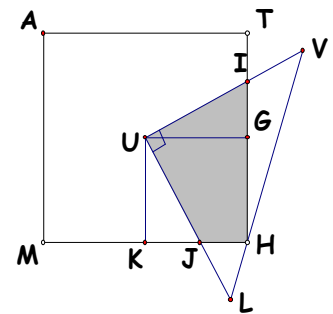
7. Label the point of intersection of the two circles inside of ABCD as E. AED is equilateral with sides of 10. Area equals $\frac{10^2\sqrt{3}}{4} = 25\sqrt{3}$. The area of each of the sectors ABE and DCE is $\frac{30}{360} \cdot \pi \cdot 10^2 = \frac{25\pi}{3}$. The shaded area is the area of ABCD minus the area of the triangle and both of the sectors.
- $$\text{Area} = 100 - 25\sqrt{3} - \frac{50\pi}{3}$$



- 8.
- | | | | |
|---|-------|-----|-------------|
| A | True | +5 | |
| B | False | -13 | |
| C | True | +23 | |
| D | False | -37 | |
| E | True | +41 | |
| F | True | +59 | Total is 78 |

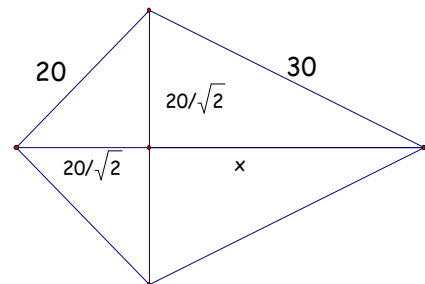
9. The x and y axis make this a right triangle with one leg equal to the y intercept of 9. To find the x intercept, solve the equation for y = 0. x = 6 Area = .5(9)(6) = 27

10. $\angle GUI \cong \angle KUJ$ because both are $90 - m\angle GUJ$. Also, both are right triangles with a leg of 5. Their areas are equal. The shaded area is simply the area of square KUGH. $5^2 = 25$.



- 11.
- | | | | | | | | |
|------------|-----|-----|-----|-----|-----|-----|---------------|
| | Day | 1 | 2 | 3 | 4 | ... | n |
| Total Nuts | | 5 | 18 | 35 | 56 | | |
| Factors | | 1·5 | 3·6 | 5·7 | 7·8 | ... | $(2n-1)(n+4)$ |
- For n = 30 $(2 \cdot 30 - 1)(30 + 4) = 2006$

12. The 90° angle has to be between the 20 cm sides because $30\sqrt{2} > 2 \cdot 20$. One diagonal is $20\sqrt{2}$. To find the other, use the Pythagorean Theorem to find $x = 10\sqrt{7}$. The area of the kite is half the product of the diagonals.
- $$\frac{1}{2}(20\sqrt{2})(10\sqrt{2} + 10\sqrt{7}) = 200 + 100\sqrt{14}$$



13. The centroid is located at the mean of the vertex coordinates.
- $$\left(\frac{-13 + 22 - 15}{3}, \frac{7 - 14 + 13}{3} \right) \text{ which is } (-2, 2)$$

14. For $\triangle ABC \sim \triangle MNP$, a scale factor of 3 times 17cm will produce a side of 51 and integer side lengths for all three sides in the not shown triangle. The largest possible side is $3(24) = 72 = K$. For $\triangle DEF \sim \triangle XYZ$, a scale factor of 5 times the 17 cm side will produce a side length of 85 and integer side lengths in the not shown triangle. Also, $\triangle DEF$ is a right triangle. The largest possible altitude is the longest leg. $5(15) = 75 = H$.
 $K + H = 72 + 75 = 147$

15. $\triangle ABG$ is a right triangle with legs of 3 and 4. BG is 5. $\triangle ABG$ is similar to $\triangle DHG$. To find DH , solve the proportion $\frac{3}{4} = \frac{DH}{1}$ to find $DH = \frac{3}{4}$ so that CH is $\frac{9}{4}$ and the area of $\triangle BCH$ is $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{9}{4} = \frac{27}{8} = 3.375$