Calculus Individual Test

Middleton Invitational 2/18/2006 *no calculator allowed*

The abbreviation NOTA denotes "None of These Answers."

1. For $f(x)=(2x-1)^5$ find $f'(1)$.

A. 2 B. 5
C. 10 D. 160 E. N OTA

2. Let $xy-xy^2=4$ for $xy\neq 0$. At the point (A, B) the value of $\frac{dy}{dx}$ is undefined.

What is the value of A • B ?

A. 0.5 B. 4
C. 8 D. 16 E. NOTA

3. For $x=1$, $\lim_{h\to 0}\frac{\sqrt{3+2x+2h}-\sqrt{3+2x}}{h}=$

A. $\frac{1}{25}$ B. $\frac{\sqrt{5}}{10}$
C. $\frac{1}{5}$ D. $\frac{\sqrt{5}}{5}$ E. NOTA

4. Two distinct differentiable functions f and g have the same derivative over all reals. That is, $f'(x)=g'(x)$ for all x. Which must be true?

A. $f(x)-g(x)$ must be constant.
B. $f'(g(x))=0$.
C. $f''(x)=g''(x)=0$.
D. $\lim_{x\to 0}f(x)=\lim_{x\to 0}g(x)$.
E. NOTA

5. For $y=\frac{\sqrt{3x+1}(4x+1)}{x^2+x}$, find $\frac{dy}{dx}$ at the point on the curve where $x=1$.

A. 2 B. $\frac{8\sqrt{5}}{3}$
C. $-\frac{13}{8}$ D. $-\frac{\sqrt{2}}{8}$
E. NOTA

6.

x	g	g'
1	4	2
2	3	4
4	2	3

If $f(x)=x^3$, and g is a differentiable function with values given in the table above and $h(x)=g(f(x))$ then find $h'(1)$.

A. 6 B. 36
C. 96 D. 128 E. NOTA

7. For f, an even differentiable function defined over all real numbers, if $f(1)=6$ and $f'(1)=2$ then find the value of $f'(-1)\bullet f(-1)$.

A. -12 B. -3
C. 3 D. 12 E. NOTA

8. For $f(x)=|3x-2|$ find $f'(0)$.

A. -3 B. -2
C. 2 D. 3 E. NOTA

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9. Use the local linearization of $f(x) = 4x^2 - x + 6$ at $x = 2$ to approximate $f(2.1)$.

   A. 20.00 B. 21.50
   C. 21.54 D. 21.56 E. NOTA

10. f is a continuous and differentiable function. Use the values in the table and a midpoint Riemann sum of four equal subdivisions to approximate the average value of f, over the interval [2, 10].

x	f(x)
1	2
2	1
3	0
4	0
5	4
6	6
7	8
8	10
9	8
10	8
11	6

   A. $\frac{25}{16}$ B. $\frac{5}{4}$

   C. 5 D. $\frac{46}{9}$

   E. NOTA

11. Find all values of c which may exist that satisfy the conclusion of the Mean Value Theorem for derivatives, given $f(x) = x^{\frac{2}{3}}$ over the interval [-1, 8].

   A. 0 B. $\frac{1}{8}$

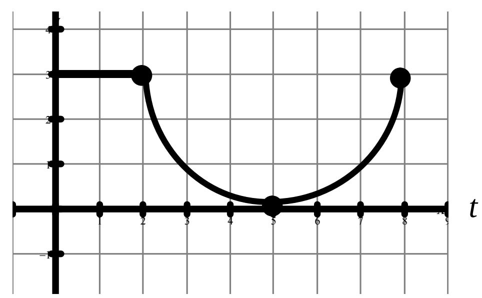
   C. $\frac{1}{3}$ D. $\frac{125}{27}$ E. NOTA

12. For a continuous and twice differentiable function f, $f'(x) < 0$ and $f''(x) > 0$ for all x. If the line tangent to f at $x = 2$ is used to approximate $f(2.1)$ then which must be true of the approximation?

   A. it overestimates $f(2.1)$
   B. it underestimates $f(2.1)$
   C. it is 0.1 less than the slope at $x = 2$
   D. it is exactly 0.1 from the value of $f(2.1)$
   E. NOTA

13. K is the positive number which has the greatest difference from its square root. Give the value of $\frac{1}{K} + \frac{1}{K^2}$.

   A. 6 B. 8
   C. 16 D. 20
   E. NOTA

14. 

The graph of f, shown above, consists of a horizontal line segment from (0, 3) to (2, 3) and a semicircle with endpoints at (2, 3) and (8, 3). $g(x) = \int_0^x f(t)dt$.

If $g(8) + g'(2) + g''(1) = A + B\pi$ then $A + B =$

   A. 9.5 B. 22.5
   C. 27.5 D. 28.5 E. NOTA

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15. For $f(x)=\begin{cases} x^{\frac{2}{3}} & \text{for } x\le 1 \\ (x-2)(x-3)-1 & \text{for } x>1 \end{cases}$,
the absolute maximum of f over [0,3] is M and the absolute minimum of f over the [0, 3] ? is m. Find M-m.

A. 1 B. $\frac{1}{2}$
C. $\frac{5}{2}$ D. $\frac{9}{4}$ E. NOTA

16. If $e^{f(x)}=4x^2+1$ then $f'(2)=$

A. $\frac{1}{17}$ B. $\frac{16}{17}$
C. $\ln 16$ D. $\ln 17$ E. N OTA

17. The derivative of f with respect to x is $(x-1)^4(x+2)(x-3)$. At which x-coordinate(s) does the graph of f have a relative maximum ?

I. $x=1$
II. $x=-2$
III. $x=3$

A. I only B. II only
C. I, II only D. I, III only
E. NOTA

18. What is the smallest initial velocity (in feet per second) needed to throw a projectile from ground level to the top of a 49-foot tall silo? Assume gravity is the only other force.

A. 98 fps B. 71 fps
C. 68 fps D. 65 fps
E. NOTA

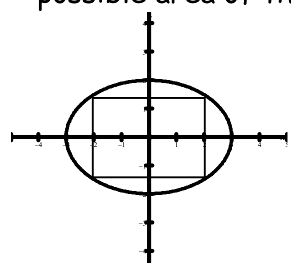
19. For $f(x)=\ln(e^{x^2-1})-\ln(e^{x-1})$ what is the value of $f'(3)$?

A. 6 B. 5
C. 4 D. 3 E. NOTA

20. The line normal to $y=Arc\tan(e^{2x})$ at $x=1$ is parallel to the line $xe^5+ex+ky=4$. Find the value of k.

A. $\frac{1-e^8}{2}$ B. $2e^3$
C. $4e^5$ D. $-2e^6$ E. NOTA

21. A rectangle, centered at the origin, is inscribed in an ellipse with equation $4x^2+9y^2=36$. What is the maximum possible area of the rectangle?



A. 27
B. $12\sqrt{3}$
C. 12
D. $4\sqrt{6}$
E. NOTA

22. The graph of $f(x)=\frac{x^2-1}{x-1}$ has which property at $x=1$?

A. f is continuous.
B. $\lim_{x\to 1} f(x)$ does not exist.
C. $\lim_{x\to 1} f(x)$ exists but $f(1)$ does not.
D. Both $\lim_{x\to 1} f(x)$ and $f(1)$ exist, but are not equal.
E. NOTA

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23. Let $g(x) = f^{-1}(x)$ and both relations, f and g are defined and differentiable over all reals.

x	$f(x)$	$f'(x)$
1	3	5
2	1	7

Give the value of $g'(1)$.

A. $\frac{1}{2}$ B. $\frac{1}{3}$
C. $\frac{1}{5}$ D. $\frac{1}{7}$ E. NOTA

24. For $y = \sin^4 x$, which is $\frac{dy}{dx}$?

A. $4\sin^3 x$
B. $2\sin(2x)\sin^2 x$
C. $-4\sin^3 x \cos x$
D. $4\cos^3 x$
E. NOTA

25. The base of a right regular hexagonal prism is changing so that the base area is increasing at 2 sq. cm per minute. The prism's height is decreasing at 1 cm per second. When the height is 10 cm and the base edge is 2 cm, find the rate at which the volume is changing, in cubic cm per **minute**.

A. $\frac{1}{3} + \frac{\sqrt{6}}{10}$
B. $20 - 6\sqrt{3}$
C. $-\frac{3}{2}\sqrt{3}$
D. $20 - 360\sqrt{3}$
E. NOTA

26. A particle travels along the x-axis with position at time t seconds ($t \geq 0$) given by $x(t) = t^3 - 3t^2 + 3t + 2$. Which statement is true about the motion of the particle at $t = 0.5$ seconds?

A. The particle is moving to the left.
B. The particle is speeding up.
C. The particle has positive acceleration.
D. The particle is at position 3.
E. NOTA

27. Let $f(x) = 2|x-3| + 4$ and $g(x) = f(|x|)$. If $h(x) = x^2 + x$, then find the sum of the values of $h'(x)$ for the x-coordinates of each critical point of the graph of g.

A. 2 B. 3
C. 7 D. 12 E. NOTA

28. The tangent line to $y = 10x - x^2$ at point P has a y-intercept of 1. If P is in quadrant I, find the y-coordinate of P.

A. 8 B. 9
C. 9.5 D. 10 E. NOTA

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29. If $f(x-2)=\cos x$ and
$g(x)=\cos(2)\sin(x)+\sin(2)\cos(x)$ and
$h(x)=\dfrac{f(x)}{g(x)}$ then find $h'(2)$.

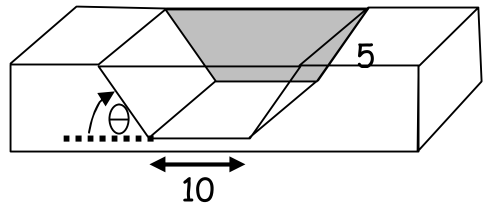
A. $\dfrac{\cos 4+\sin 4}{\sin^2 4}$

B. $\dfrac{1}{5\cos 4}$

C. $-\csc^2 4$

D. $\dfrac{-\tan 3}{2\sin 8}$

E. NOTA



30. A drainage trough has the shape of a trapezoidal prism as shown. The smaller base of the trapezoid (which is not drawn to scale) is 10 feet and the congruent legs of the trapezoid are 5 feet each. The sides of the trough will have equal slopes and the acute angle of the sides off the horizontal is θ. Find $\cos\theta$ so that the area of the trapezoidal cross section (shaded) is maximized.

A. $\dfrac{\sqrt{3}-1}{2}$

B. $\dfrac{\sqrt{2}+1}{4}$

C. $\dfrac{\sqrt{3}}{4}$

D. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

E. NOTA

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Solutions:

1. **C.** Using the chain rule, $f' = 5(2x-1)^4 \bullet 2$ and at x=1, f'=10.

2. **C**. $\frac{dy}{dx} = \frac{y^2 - y}{x - 2yx}$ this is undefined at x=0 (no point on the curve for this, since xy$\neq$0) or y=1/2. Substitute into the curve to get x=16, and AB = $\frac{1}{2} \bullet 16 = 8$.

3. **D**. The definition of the derivative gives $\frac{d}{dx}(3+2x)^{\frac{1}{2}} = \frac{1}{2}(3+2x)^{\frac{-1}{2}} \bullet 2 =$ evaluated at x=1 gives $5^{\frac{-1}{2}} = \frac{1}{\sqrt{5}}$ which gives choice D.

4. **A**. The slope of the tangent line is the same at any point, so the shape of the graph is the same, but the graphs differ by a constant.

5. **C.**

$$\ln y = \tfrac{1}{2}\ln(3x+1) + \ln(4x+1) - \ln(x^2 + x)$$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2} \bullet \frac{3}{3x+1} + \frac{4}{4x+1} - \frac{2x+1}{x^2+x}$. At x=1, y=5, and so $\frac{dy}{dx} = (\frac{3}{8} + \frac{4}{5} - \frac{3}{3}) \bullet 5$. Simplify to -13/8, which is choice **C.**

6. **A**. Using the chain rule, $h'(1) = g'(f(1)) \bullet f'(1) = g'(1) \bullet 3 = 2(3) = 6$.

7. **A**. An even function has the property that f(-x)= f(x) so f(-1)= 6. The slopes are opposites at opposite x's, so f'(-1) = -2. The product is -12.

8. **A**. The corner of the graph is at x=2/3 so at x=0 we have a negative 3 slope.

9. **B**. The tangent line at x=2 has equation (y-20)=15(x-2) and letting x=2.1 gives y=21.5.

10. **C.** The integral from 2 to 10 is 2(f(3)+f(5)+f(7)+f(9)) = 40. The average value is 1/8 of this, which is 5.

11. **E.** The graph is not differentiable so the Mean Value Theorem does not apply.

12. **B**. Since f is concave up, the tangent line is below the curve.

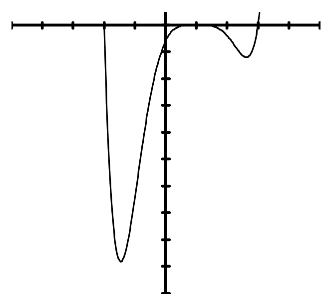
13. **D**. Maximize $\sqrt{x} - x$ by setting $\frac{1}{2}x^{-1/2} - 1 = 0$ to get x=1/4. 4+16=20

14. **B**. The area below the graph is 3(8)-the area of the semicircle. So $\int_0^8 f(t)dt = 24 - 4.5\pi$.
The value of g'(2)=f(2) (by the FTC) which is 3. The value of g"(1)=f'(1)=0. The sum of A and B is therefore 27-4.5 = 22.5.

15. **D**. At x=0 and x=1 f' is undefined. At x=5/2 f'=0. So we check values at endpoints (since we are looking for abs.val and get m= -5/4 and M=1. 1-(-5/4)=9/4.

16. **B**. $f(x) = \ln(4x^2+1)$ so $f'(x) = \frac{8x}{4x^2+1}$ and at x=2, this equals 16/17.

17. **B**. The graph of f' will "bounce" at the x-intercept x=1. A sixth power graph with intercepts 1, -2 and 3 will look like the graph below. You can check intervals for positive and negative values of f', or you can use your knowledge of the graph to find the derivative goes from positive then negative at x= -2.



18. **E.** Using $-16t^2 + v_0 t = position$, get the derivative and set=0 to get $t = \frac{v_0}{32}$. Use the position function, this t, and set = 49 to get **56** fps.

19. **B.** Since lnx and e^x are inverse functions f can be simplified to $x^2 - 1 - (x-1) = x^2 - x$ and its derivative is 2x-1. At x=3 this gives 5, choice B.

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20. B. The slope of the line given is $\frac{e^5+e}{-k}$ and the derivative of the function at x=1 is $\frac{2e^2}{1+e^4}$ and its perpendicular slope is $\frac{1+e^4}{-2e^2}$. Setting this equal to $\frac{e^5+e}{-k}$ gives k is choice B.

21. C. For point (x,y) in quadrant I, the area of the rectangle is 2x times 2y, or $4x\sqrt{9-\frac{4}{9}x^2}$ and the derivative of this is

$$4\sqrt{9-\frac{4}{9}x^2}+\frac{1}{2}(9-\frac{4}{9}x^2)^{-½}(-4x/9)(4x)$$

and getting a common denominator gives $4(9-\frac{4}{9}x^2)-\frac{16x^2}{9}=0$ (numerator of the fraction) for x= $\frac{3}{\sqrt{2}}$. The area is then

$$4(\sqrt{2})(\frac{3}{\sqrt{2}})=12$$

22. C. The graph has a removable discontinuity (a hole) at x=1, and so the limit exists. The point does not. Choice C.

23. D. The graph of g goes through (1, 2) and the graph of f goes through (2, 1). The deriv. of g at x=1 is the reciprocal of the derivative of f at x=2. Choice D gives 1/7.

24. B. $\frac{d}{dx}(\sin x)^4=4(\sin x)^3(\cos x)=$

$4(\sin x\cos x)\sin^2 x=2\sin(2x)\sin^2 x$.

25. D. The volume of a prism is Bh where B denotes the area of the base. The area of a regular hexagon is $1.5(edge)^2\sqrt{3}$.

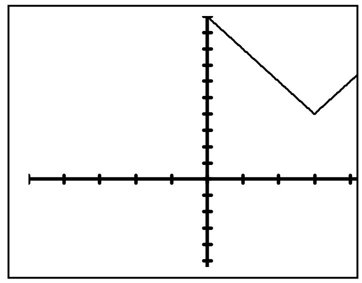
$\frac{dV}{dt}=\frac{dB}{dt}(h)+\frac{dh}{dt}B$ and $2(10)+-60(6\sqrt{3})$

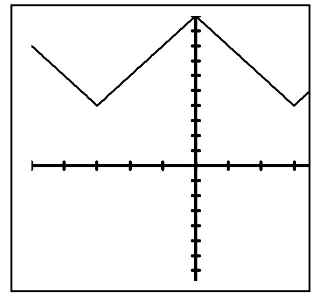
dv/dt=20-360 $\sqrt{3}$ cu cm/min.

26. E. At t=0.5 the velocity is positive and the acceleration is negative which means the particle is slowing down.

27. B The graph of the original f is shown below.

and the graph of f(|x|) will reflect over the y-axis to get the graph shown below.



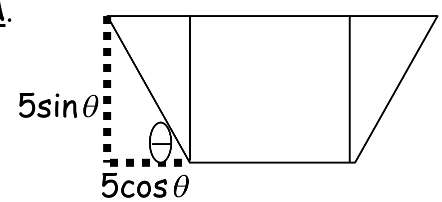


The graph has three critical points, where the derivative is undefined. x=-3, 0, 3. The derivative of h at each of these points added gives (2x+1): -5, 1, 7 respectively. Sum=3.

28. B. The slope of the line through the two points (0, 1) and $(x, 10x-x^2)$ should be equal to the slope of the tangent line so $\frac{10x-x^2-1}{x-0}=f'=10-2x$. Solving gives x= ±1 gives the point in quadrant I has x=1, and y=10-1^2=9. Choice B.

29. C. $f(x)=\cos(x+2); g(x)=\sin(x+2)$ and $h(x)=\cot(x+2);\ h'(x)=-\csc^2(x+2)$

30. A.



The bases of the trapezoid are 10 and 10+$5\cos\theta+5\cos\theta$. The height of the trapezoid is $5\sin\theta$. The area of the trap is $\frac{1}{2}(5\sin\theta)(20+10\cos\theta)$. Distributing, and deriving:

$50\sin\theta+25\sin\theta\cos\theta=50\sin\theta+12.5\sin(2\theta)$.

$A'=50\cos\theta+25\cos(2\theta)$= 0. Divide by 25.

$2\cos\theta+1(2\cos^2\theta-1)$=0. Using the quad.

formula gives $\cos\theta = \frac{-2 \pm \sqrt{4-4(1)(-2)}}{4}$. We do not use the negative value for cosine since theta is acute.

Question #1 Calculus Team

Let A be the set of value(s) of c which satisfies the conclusion of the Mean Value Theorem (for derivatives) for $f(x)=x^3-2x^2+x$ on the interval [0, 1].

Let B be the set of value(s) of $f''(x)$ when $f'(x)=1$ for $f(x)=x^3-2x^2+x$, where $x>0$.

Give the sum of all members of $A\cup B$.

Question #2 Calculus Team

Let $f(x)=-2x^3+2\sqrt{x}+8x+3$

Let A be the slope of the line normal to f at the point (1, 11).

Let B be the slope of the line tangent to f at $x=4$.

Let C be the slope of the inverse $f^{-1}(x)$ at the point on $f^{-1}(x)$ where $x=11$.

Give the value of $A \bullet B \bullet C$.

Calculus Team Question #3

Let A be the value of $\lim_{x \to 9} \frac{x-9}{x-\sqrt{3}}$.

If the limit does not exist, let $A = 10$.

Let B be the value of

$\lim_{h \to 0} \frac{(3+h)^3 - 2(3+h)^2 - (27) + 2(9)}{h}$.

If the limit does not exist, then let $B = 20$.

Let C and D be the values for which

$f(x) = \begin{cases} Cx^2 + Dx + 1 & for \quad x \le 2 \\ 13 - Cx & for \quad x > 2 \end{cases}$ is both

continuous and differentiable.

Give the value of $A + B + C + D$.

Calculus Team Question #4

For $y = \sec^4(2x^2)$ at let $\frac{dy}{dx} = Ax(\sec(2x^2))^P \tan(2x^2)$

Let B be the value of x where the graph of g has a relative minimum, given that

$g'(x) = (x-2)^2(x+3)(4-x)$.

Let C be the value of $\frac{d^2y}{dx^2}$ for $t = 1$ if

$y = 2t$ and $x = t^3$.

Give the value of $\frac{9 \bullet A \bullet B \bullet C \bullet P}{16}$.

Calculus Team Question #5

For $f(x) = x^2 - \frac{1}{x}$, let S be the set of integer $x-$coordinates for which the graph of f is concave up. Do not include inflection points.

Let T be the set of integer $x-$coordinates for which the graph of f is increasing.

Let Q be the values of $T \cap S$ which are in the domain of f and which satisfy the inequality $|x| < 5$.

Give the members, in order, of Q.

Calculus Team Question #6

$f(x) = \left| \, |x-6| - 6 \, \right|$

Let A be the value of $f'(-1)$.

Let B be the set of x value(s) for which there is a critical point on the graph of f.

Let C be the maximum value of f over the interval [0, 6].

Give the sum of A, C and all members of set B.

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Calculus Team Question #7

A single term for the term a_n sequence $-5,\ 9,\ -5,\ 9,\ -5,\ 9,\ \ldots$ given that for $n=1$ the first term is a_1, is $A(-1)^n + B$.

Let C the value of $\frac{d^2y}{dx^2}$ at the point $(6,-8)$ on the circle with equation $x^2 + y^2 = 100$.

Give the product $64 \bullet A \bullet B \bullet C$.

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Calculus Team Question #8

Consider the velocity of a particle, moving along the x-axis $v(t) = t^2 - 6t + 5$ in units per minute for $t \geq 0$ minutes.

Let ($\boldsymbol{S, V}$) be the second complete interval of t when the particle is slowing down.

Let $(\boldsymbol{T, W})$ be the complete interval of t when the particle is moving to the left.

Let value $\boldsymbol{U}$ units per minute^2 be the maximum acceleration of the particle over the time interval [0, 4].

Give the sum $S + V + T + W + U$.

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Calculus Team Question #9

Using the fact that, $\sqrt{100} = 10$, and using differentials to approximate $\sqrt{96}$ gives value $\frac{A}{5}$.

Using the fact that $\sqrt[3]{27} = 3$, and using differentials to approximate $\sqrt[3]{28}$ gives value $\frac{B}{27}$.

Using the fact that $\frac{1}{\sqrt{9}} = \frac{1}{3}$ and using differentials to approximate $\frac{1}{\sqrt{9.2}}$ gives a value of $\frac{1}{3} - \frac{1}{C}$.

Give the value of $A + B + C$.

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Calculus Team Question #10

The graph of a continuous function f has a horizontal normal line at the point (1, 4) on the curve. The equation of the tangent line at that point is $Ax + By = 8$.

If $G(x) = \int_{1}^{2x} \frac{1}{1+t^2}\,dt$ then let C = $G'(2)$.

Give the value of $A \bullet C + B$.

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Calculus Team Question #11

The volume of a cube is increasing at the rate of
20 cubic cm per second. When the edge is 10 cm...

its surface area is increasing at S square cm per second,

its diagonal is increasing at D cm per second,

its shadow is a parallelogram with base and height equal to the length of the cube's edge. The rate that the area of the parallelogram is changing is P sq. cm.

Give the value of $S \bullet D \bullet P$.

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Calculus Team Question #12

An isosceles triangle has two sides with length 8 and included angle θ. If the legs stay constant and θ is increasing at $\frac{\pi}{180}$ radians per minute, then

let A be the rate of change of the area of the triangle when θ is $\frac{\pi}{3}$ radians

let B be the distance from the vertex of the triangle to the base, when θ is $\frac{\pi}{3}$ radians.

Find the value of $\frac{A \bullet B}{\pi\sqrt{3}}$.

Calculus Team Question #13

$f(x)=\dfrac{3x\sqrt{x+2}}{x-1}$ and $f'(2)=A$

$g(x)=4\sin(3-x)\cos(3-x)$ and $g'(2)=B\cos C$

$h(x)=e\bullet e^{1-x}$ and $h'(2)=D$

Give the value of $(A\bullet B)+C+D$.

Calculus Team Question #14

$f(x)=6x^2$ and $g(x)=5x-1$ intersect at the points (A, B) and (C, D), for $A<C$.

Let E be the value of $f'(A)$ and let F be the value of $g'(C)$.

Give the value of $\dfrac{E\bullet F}{A\bullet C}$.

Calculus Team Question #15

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	8	2	-1	4
2	4	5	-2	5
-1	A	B	C	D

f and g are continuous and twice-differentiable functions, both defined over all reals. $h(x) = f(g(x))$.

f is an even function, and g is an odd function.

Let P be the value of $h'(1)$.

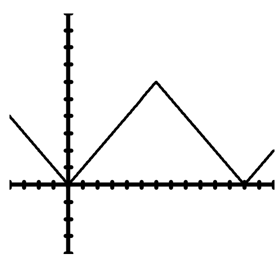
Give the sum A+B+C+D+P.

Solutions: Calculus Team SOLUTIONS Middleton Invitational 2-18-2006

1. f(0)=0 and f(1)=0 so set f'=0 and get x=1/3 and x=1. But MVT does not include endpoints so A=1/3. f'=1 x=0 and x=4/3. f"= -4 and 4 so B={ 4}. Sum = **13/3**.
2. Find f'(1)= 3 to get the normal line has slope -1/3 = A. B= f'(4)=-6(16)+0.5+8= -175/2. C is 1/f'(1) which is 1/3. The product is **175/18**.
3. A=0, and B is 15 by getting the derivative of $x^3 - 2x^2$ at x=3. 4C+2D+1=13-2C by using x=2 into both parts, to ensure continuity. Do the same for both derivatives to ensure differentiability, and get 2Cx+D=-C and 4C+D= -C, and solving both gives C= -3 and D=15. So 0+15+-3+15 = **27**.
4. dy/dx = $4x(\sec(2x^2))^3(\sec(2x^2)\tan(2x^2))(4x)$ so A=16 and P=4. Sketching g' by use of knowledge of a quartic graph and double roots gives f' is negative until -3, positive to 2 (where it is tangent to the x-axis) and then positive to 4, then negative. So x= -3 is a rel. min. B= -3. y= $2x^{\frac{1}{3}}$ so dy/dx = $\frac{2}{3}x^{-2/3}$ and the second deriv. is $\frac{-4}{9}x^{\frac{-5}{3}}$ and at x=1 equals -4/9.

So $\dfrac{9 \bullet 16 \bullet -3 \bullet 4 \bullet \frac{-4}{9}}{16}$ = **48**.

5. S is $-1, \pm 2, \pm 3, \pm 4, \ldots$ and T is 1, 2, 3, 4, …. Q is then **2, 3, 4**.
6. The graph of $y = |x-6|$ is a "V" with vertex at (6,0). Lower this 6 units and we get a graph with x-intercepts at 0 and 12 and vertex at (6, -6). Reflect this up, for $f(x) = \left| \, |x-6| - 6 \, \right|$ and we get which has f'(-1) = -1 and critical points at x=0, 6, 12. C is 6. -1+0+6+12+6 = **23**.



7. A=7 and B=2 which we can get from B-A= -5 and B+A=9. C: dy/dx= -x/y = 3/4 and y" = (-y+y'(x))/yy = (8+3/4 (6))/64 = 25/128. So 64 times 25/128 times 7 times 2 = **175**
8. The graph is a concave-up parabola with roots at 1 and 5. (S,V)=(3, 5), when acceleration and velocity have different signs. (T, W)= (1, 5) and U=6 since a=2t-6 which has max at the right endpoint, x=6. y=6. The sum is 3+5+1+5+2 = **16**.
9. A=49, by $\frac{1}{2}(100)^{-1/2}(-4)+10$ to get 49/5. B is 82 by $\frac{1}{3}(27)^{-2/3}(1)+3=\frac{82}{27}$.

C=270 by $-\frac{1}{2}x^{-3/2}(\frac{1}{5})+\frac{1}{3}$. A+B+C = 49+82+270 = **401**.

10. A horizontal normal means a vertical tangent. So x=1 is the equation, and 8x+0Y=8 gives A=8 and B=0. By the FTC, G'(x)= $\frac{1}{1+(2x)^2}(2)=\frac{2}{17}$. The sum is **16/17**.
11. V= x^3, $\frac{dv}{dt} = 3x^2\frac{dx}{dt}$. So 20=3(100)dx gives dx= 1/15. A= $6x^2$, $\frac{dA}{dt} = 12x\frac{dx}{dt}$. so

S=12(10)(1/15) to get S=8. Diagonal is $x\sqrt{3}$ so D= $\frac{\sqrt{3}}{15}$ and Area =

x^2, $2x\frac{dx}{dt}=\frac{20}{15}\sqrt{3}=\frac{4}{3}\sqrt{3}$ =P. $8\bullet\frac{\sqrt{3}}{15}\bullet\frac{4}{3}=\boxed{\frac{32}{45}\sqrt{3}}$.

12. $A=\frac{1}{2}abSinC=32\sin\theta$ so $\frac{dA}{dt}=32\cos\theta\frac{d\theta}{dt}=32(\frac{1}{2})(\frac{\pi}{180})$ = $\frac{4}{45}\pi$.

B = $\cos\left(\frac{1}{2}\theta\right)=\frac{C}{8}$ gives C= $4\sqrt{3}$. $\frac{AB}{\pi\sqrt{3}}=\boxed{\frac{16}{45}}$

13. A= -9/2: $\ln y=\ln(3x)+\frac{1}{2}\ln(x+2)-\ln(x-1)$ so $\frac{1}{y}\frac{dy}{dx}=\frac{1}{x}+\frac{1}{2(x+2)}-\frac{1}{x-1}$

and since y=12, y'=(1/2+1/8-1) times 12. g= 2sin(6-2x) by the double angle formula and g'=-2(2)cos(6-2x) = -4cos2 so B= -4 and C=2. h'= $-e^{2-x}$ and so D= -1. AB+C+D = -9+2+-1 for answer **19**.

14. A=1/3 and B=2/3 and C=1/2 and D=3/2. E=4 and F=5 for an answer of 20 divided by 1/6 which gives **120**.

15. A=8, B= -2, C=1, D= 4 by def. of even and odd functions and properties of their derivatives. h'(1) = f'(g(1)) times g'(1) = f'(2) times 4 = -2 times 4 = -8.
8 + -2 + 1 + 4 + -8 gives **3**.

Sponsor's Copy Middleton Invitational Calculus 2-18-2006

1. Let A be the set of value(s) of c which satisfies the conclusion of the Mean Value Theorem (for derivatives) for $f(x) = x^3 - 2x^2 + x$ on the interval [0, 1].

   Let B be the set of value(s) of $f''(x)$ when $f'(x) = 1$ for $f(x) = x^3 - 2x^2 + x$, where $x > 0$.

   Give the sum of all members of $A \cup B$.

2. Let $f(x) = -2x^3 + 2\sqrt{x} + 8x + 3$

   Let A be the slope of the line normal to f at the point (1, 11).

   Let B be the slope of the line tangent to f at $x = 4$.

   Let C be the slope of the inverse $f^{-1}(x)$ at the point on $f^{-1}(x)$ where $x = 11$.

   Give the value of $A \bullet B \bullet C$.

3. Let A be the value of $\lim_{x \to 9} \dfrac{x-9}{x-\sqrt{3}}$. If the limit does not exist, let $A = 10$.

   Let B be the value of $\lim_{h \to 0} \dfrac{(3+h)^3 - 2(3+h)^2 - (27) + 2(9)}{h}$. If the limit does not exist, then let $B = 20$.

   Let C and D be the values for which $f(x) = \begin{cases} Cx^2 + Dx + 1 & \text{for } x \le 2 \\ 13 - Cx & \text{for } x > 2 \end{cases}$ is both continuous and differentiable.

   Give the value of $A + B + C + D$.

4. For $y = \sec^4(2x^2)$ at let $\dfrac{dy}{dx} = Ax(\sec(2x^2))^P \tan(2x^2)$

   Let B be the value of x where the graph of g has a relative minimum, given that $g'(x) = (x-2)^2(x+3)(4-x)$.

   Let C be the value of $\dfrac{d^2y}{dx^2}$ for $t = 1$ if $y = 2t$ and $x = t^3$.

   Give the value of $\dfrac{9 \bullet A \bullet B \bullet C \bullet P}{16}$.

5. For $f(x) = x^2 - \dfrac{1}{x}$, let S be the set of integers for which the graph of f is concave up. Do not include inflection points.

   Let T be the set of integers for which the graph of f is increasing.

   Let Q be the values of $T \cap S$ which are in the domain of f and which satisfy the inequality $|x| < 5$.

   Give the members, in order, of Q.

6. $f(x) = \left| |x-6| - 6 \right|$

Let A be the value of $f'(-1)$.

Let B be the set of x value(s) for which there is a critical point on the graph of f.

Let C be the maximum value of f over the interval [0, 6].

Give the sum of A, C and all members of set B.

7. A single term for the term a_n sequence $-5,\ 9,\ -5,\ 9,\ -5,\ 9,\ ...$ given that for $n=1$ the first term is a_1, is $A(-1)^n + B$.

Let C the value of $\frac{d^2y}{dx^2}$ at the point $(6,-8)$ on the circle with equation $x^2 + y^2 = 100$.

Give the product $64 \bullet A \bullet B \bullet C$.

8. Consider the velocity of a particle, moving along the x-axis $v(t) = t^2 - 6t + 5$ in units per minute for $t \geq 0$ minutes.

Let ($\boldsymbol{S}$, $\boldsymbol{V}$) be the second complete interval of t when the particle is slowing down.

Let ($\boldsymbol{T}$, $\boldsymbol{W}$) be the complete interval of t when the particle is moving to the left.

Let value $\boldsymbol{U}$ units per minute^2 be the maximum acceleration of the particle over the time interval [0, 4].

Give the sum $S + V + T + W + U$.

9. Using differentials to approximate $\sqrt{96}$ gives value $\frac{A}{5}$.

Using differentials to approximate $\sqrt[3]{28}$ gives value $\frac{B}{27}$.

Using differentials to approximate $\frac{1}{\sqrt{9.2}}$ gives a value of $\frac{1}{3} - \frac{1}{C}$.

Give the value of $A + B + C$.

10. The graph of a continuous function f has a horizontal normal line at the point (1, 4) on the curve. The equation of the tangent line at that point is $Ax + By = 8$.

If $G(x) = \int_1^{2x} \frac{1}{1+t^2}\,dt$ then let C the value of $G'(2)$.

Give the value of $A \bullet C + B$.

11. The volume of a cube is increasing at the rate of 20 cubic cm per second. When the edge is 10 cm…

its surface area is increasing at S square cm per second,

its diagonal is increasing at D cm per second,

its shadow is a parallelogram with base and height equal to the length of the cube's edge. The rate that the area of the parallelogram is changing is P sq. cm.

Give the value of $S \bullet D \bullet P$.

12. An isosceles triangle has two sides with length 8 and included angle θ. If the legs stay constant and θ is increasing at $\frac{\pi}{180}$ radians per minute, then

let A be the rate of change of the area of the triangle when θ is $\frac{\pi}{3}$,

let B be the distance from the vertex of the triangle to the base, when θ is $\frac{\pi}{3}$.

Find the value of $\frac{A \bullet B}{\pi\sqrt{3}}$.

13. $f(x) = \frac{3x\sqrt{x+2}}{x-1}$ and $f'(2) = A$

$g(x) = 4\sin(3-x)\cos(3-x)$ and $g'(2) = B\cos C$

$h(x) = e \bullet e^{1-x}$ and $h'(2) = D$

Give the value of $(A \bullet B) + C + D$.

14. $f(x) = 6x^2$ and $g(x) = 5x - 1$ intersects at the points (A, B) and (C, D), for $A < C$.

Let E be the value of $f'(A)$ and let F be the value of $g'(C)$.

Give the value of $\frac{E \bullet F}{A \bullet C}$.

15.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	8	2	-1	4
2	4	5	-2	5
-1	A	B	C	D

f and g are continuous and twice-differentiable functions, both defined over all reals.

$h(x) = f(g(x))$.

f is an even function, and g is an odd function.

Let P be the value of $h'(1)$.

Give the sum A+B+C+D+P.

ANSWERS TO CALCULUS BOWL

Middleton Invitational 2006

1. $\frac{13}{3}$

2. $\frac{175}{18}$

3. 27

4. 48

5. 2, 3, 4

6. 23

7. 175

8. 16

9. 401

10. $\frac{16}{17}$

11. $\frac{32}{45}\sqrt{3}$

12. $\frac{16}{45}$

13. 19

14. 120

15. 3