Alpha Open Test

NO CALCULATOR! Middleton Invitational 2/18/2006

The abbreviation NOTA denotes None of These Answers.

1. How many zeros are at the end of the expansion of 126! ?

   A. 25 B. 30
   C. 31 D. 32
   E. NOTA

2. Let $x=\sqrt{132+\sqrt{132+\sqrt{132+\sqrt{...}}}}$ and let $y=\sqrt{132-\sqrt{132-\sqrt{132-\sqrt{...}}}}$ then which is the value of $x+y$.

   A. 0 B. 23
   C. 24 D. 26
   E. NOTA

3. Evaluate $Sin(Cos^{-1}\frac{1}{9})$.

   A. $\frac{8}{9}$ B. $\frac{9}{4\sqrt{5}}$
   C. $\frac{4\sqrt{5}}{9}$ D. $4\sqrt{5}$
   E. NOTA

4. What is the amplitude times the period of the graph of $y=13\sin x\cos x$?

   A. 26π B. 13π
   C. $\frac{13}{2}\pi$ D. $\frac{\sqrt{13}}{2}\pi$
   E. NOTA

5. Simplify $\frac{\cot\theta}{\cos\theta-2\sin^2\theta\cos\theta}$ completely for $0<\theta<\frac{\pi}{4}$?

   A. $\frac{1}{\sin(3\theta)}$
   B. $\frac{1}{\sin\theta cos(2\theta)}$
   C. $\frac{1}{\cos\theta\sin\theta}$
   D. $\frac{\sin\theta}{\cos^2\theta\cos(2\theta)}$
   E. NOTA

6. How many distinct triangles ABC can be made with $m\angle A=30^\circ$, $BC=2$, $AC=3$?

   A. 2 B. 1
   C. 0 D. infinitely many
   E. NOTA

7. What is the amplitude of the graph of $y=-2\sin(x)+7\cos(x)$?

   A. 9 B. 11
   C. $2\sqrt{7}$ D. $\sqrt{53}$
   E. NOTA

8. What value of k will make the vectors $\langle 2,-7,k\rangle$ and $\langle 3,2,6\rangle$ orthogonal?

   A. -2 B. $-\frac{4}{3}$
   C. $\frac{4}{3}$ D. $\frac{7}{6}$ E. NOTA

9. In triangle MHS, with MH=3, MS=4 and HS=2, determine the area of MHS.

A. 12 B. $\frac{3\sqrt{15}}{4}$

C. $\frac{3\sqrt{15}}{2}$ D. $\frac{5\sqrt{5}}{4}$

E. NOTA

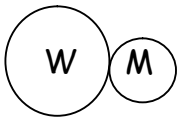
10. If the base-ten number 60 is written in base two, what is the sum of the digits?

A. 4 B. 5

C. 6 D. 7

E. NOTA

11. If circle W has a radius of 6 times the radius of circle M, and the circles are externally tangent to each other, then how many revolutions will M make if W makes one revolution?



A. 6 B. 5

C. 4 D 3

E. NOTA

12. Evaluate i^{2006} for $i=\sqrt{-1}$.

A. i B. 1

C. $-i$ D. -1

E. NOTA

13. If $\frac{|3i+4|}{1-i}=a+bi$ then $a+b=$

A. 5 B. 5.5

C. 7 D. 7.5

E. NOTA

14. Find the sum of the rational roots of the equation $x^3-x^2-3x-1=0$.

A. 2 B. 1

C. -1 D. -2

E. NOTA

15. The graph of the polar equation $r=\frac{3}{3-4\cos\theta}$ is a

A. ellipse B. hyperbola

C. lemniscate D. limaçon

E. NOTA

16. If x and y are real numbers then what is the domain of $y=\sqrt{3-x^2}$?

A. $|x|\le\sqrt{3}$ B. $|x|<\sqrt{3}$

C. $|x|\ge\sqrt{3}$ D. all reals

E. NOTA

17. Find the sum of the positive integral factors of 10.

A. 7 B. 8

C. 10 D. 18

E. NOTA

18. Evaluate $\left(\log_2 9\right)\bullet\left(\log_3\sqrt{8}\right)$.

A. 1 B. 1.5

C. 2 D. 6

E. NOTA

19. If $J+U=4$ and $J^2+U^2=20$ then give the value of $2 \bullet J \bullet U$.

A. 16 B. 12
C. -4 D. $2i$
E. NOTA

20. Express $\sqrt{396}$ in simplest radical form, $a\sqrt{b}$ and then give $a+b$.

A. 11 B. 17
C. 19 D. 20
E. NOTA

21. $4(\cos 60^\circ)^{-\frac{3}{2}} =$

A. $8\sqrt{2}$ B. $\sqrt[3]{3}$
C. $\frac{\sqrt{2}}{2}$ D. $4\sqrt{2}$
E. NOTA

22. Jorge walked 12 miles uphill in 30 minutes, and then 12 miles downhill at a rate of 4 miles per hour. What was Jorge's average speed in miles per hour for the entire trip?

A. 5 mph B. 14 mph
C. $6\frac{6}{7}$ mph D. $4\frac{4}{5}$ mph
E. NOTA

23. Will walked 8 miles, downhill, at 4 miles per hour, then completed the 24-mile-total downhill trip by falling, laying still for 10 minutes, then rolling the rest of the trip. If his average rate for the downhill journey (walking, lying and rolling) was 6 miles per hour, find the rate in miles per hour that Will rolled.

A. $13\frac{5}{6}$ B. $8\frac{8}{11}$
C. $2\frac{1}{3}$ D. $2\frac{1}{6}$
E. NOTA

24. Let A be the smallest prime greater than 49, and B be the greatest prime less than 100, and C be the smallest whole number, then give the value of A+B+C.

A. 149 B. 150
C. 151 D. 152
E. NOTA

25. The graph of $y=\frac{x^2+1}{x-1}$ approaches the line $y=mx+b$ as x approaches positive infinity. What is the value of $3m+b$?

A. 0 B. 4
C. 5 D. 6
E. NOTA

26. The graphs of $r = 4\cos\theta$ and $r = 2$ meet at the points R and S. Give the length of the minor arc $\widehat{RS}$ on the graph of $r = 4\cos\theta$.

A. $\frac{4\pi}{3}$ B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$ D. $\frac{\pi}{8}$

E. NOTA

27. The three third-roots of $(8i)$ are ai, $b+ci$ and $b-ci$. Give the value of a^2+c^2.

A. 6 B. 5

C. 4 D. 3

E. NOTA

28. A regular octagon with side length 20 is inscribed in a circle. Let $x = \sin 22.5^o$, $y = \cos 22.5^o$ and $z = \tan 22.5^o$. Which is an expression for the area outside of the octagon and inside of the circle?

A. $\frac{50\pi x^2 - 800}{y}$ B. $\frac{100\pi - 800xy}{x^2}$

C. $\frac{100\pi x - 400y}{z}$ D. $100\pi x^2 - 800z$

E. NOTA

29. Which is an expression for $\cos(Arc\cos(x) - Arc\sin(x))$ for $0 < x < 1$?

A. $\frac{1}{x^2 - x^4}$ B. $x^2 - x^4$

C. $2x\sqrt{1-x^2}$ D. $\frac{\sqrt{1-x^2}}{2x}$

E. NOTA

30. If $\sqrt{2x+1} - \sqrt{x-3} = 4$ for real value of x, then what is the value of $\sqrt{100-x}$?

A. 16 B. 8

C. 6 D. 4

E. NOTA

Alpha Open Test

NO CALCULATOR! Middleton Invitational 2/18/2006

Solutions:

1. Divide by 5, by 25 and by 125. Add the results and you get 31. **C**.
2. Short cut: 132 = 12(11) so the expression with the "addition" is the larger and the "subtraction is the smaller. x=12 and y=11. Sum 23. **B**.
3. Draw a right triangle in quadrant I and the x=1, the hypotenuse is 9 so y= $\sqrt{80}$. The sin is then $\frac{4\sqrt{5}}{9}$ which is **C**.
4. The expression is equal to $\frac{13}{2}\sin(2x)$ by the double-angle sine property. So the amp=6.5 and the period is 2π divided by the coefficient 2, to give the answer is $\frac{13}{2}\bullet\pi$ = **C**.
5. $\frac{\cos\theta}{\sin\theta}\bullet\frac{1}{\cos\theta(\cos 2\theta)}$ by factoring out a cosine from the bottom right and using the cosine double-angle rule. simplify to choice **B**.
6. AC times sinA gives 1.5, and since 2 (side opposite A) is more than 1.5 and less than 3, there are two solutions. **A**.
7. $\sqrt{2^2+7^2}$ = **D**.
8. The dot product must be 0: 6-14+6k=0 so k=4/3. Answer **C**.
9. Using Heron's formula: s=9/2

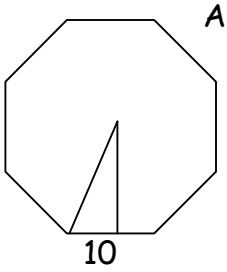
$$\sqrt{\frac{9}{2}\left(\frac{9}{2}-\frac{6}{2}\right)\left(\frac{9}{2}-\frac{8}{2}\right)\left(\frac{9}{2}-\frac{4}{2}\right)}=\sqrt{\frac{9(3)(5)}{16}}$$

$=\frac{3\sqrt{15}}{4}$ = **B**.

10. In base 2, we get 111100 for a digit sum of 4. Choice **A**.
11. The circumferences will be in the same ratio as the radii. Answer **A**.
12. $i^{2006}=i^2=-1$. Choice **D**.
13. The numerator is 5, so $\frac{5}{1-i}=\frac{5(1+i)}{2}$ and a+b = 5/2+ 5/2 = 5. Choice **A**.
14. -1 is a root. Divide by this and you get that the other two roots are irrational. So the sum of the rational roots is -1. **C**.
15. Change to rectangular form, or use the rule of coefficients. If you do the former, you get $3r-4r\cos\theta=3$ and $3\sqrt{x^2+y^2}-4x=3$ then $3\sqrt{x^2+y^2}=3+4x$ which we square to get $9(x^2+y^2)=14x^2+24x+9$ which is a hyperbola. Choice **B**.
16. $|x|\le\sqrt{3}$ since $3-x^2\ge 0$. Choice **A**.
17. 1+2+5+10 = **D**.
18. $\frac{2\log 3}{\log 2}\bullet\frac{\frac{1}{2}\bullet 3\bullet\log 2}{\log 3}=3$. Choice **E**.
19. Square the first equation to get $J^2+2JU+U^2=16$. Substitute the 2nd equation and subtract to get 2JU = -4. Choice **C**.
20. $6\sqrt{11}$ gives 6+11 is 17. Choice **B**.
21. $4(\cos 60^\circ)^{-\frac{3}{2}}=4\left(\frac{1}{2}\right)^{-\frac{3}{2}}=4(2)^{\frac{3}{2}}=4\sqrt{8}$ which is $8\sqrt{2}$ or choice **A**.
22. total distance /total time = 24/(.5+3) = 24/3.5 = 240/35 = 48/7 = **C**.
23. distance/time = 24/(2+1/6+x)=6 solves to time rolling is 11/6. So rate is 16 miles rollling divided by time 11/6 gives answer **B**.
24. 53+97+0 = 150. **B**.
25. Divide to get $y=x+1+\frac{2}{x-1}$ and the slant asymptote is y=x+1 for 3m+b=4. **B**.

26. Set the equations equal and we get $\cos\theta = \frac{1}{2}$ and they meet when $\theta = \frac{\pi}{3}$ and $\theta = -\frac{\pi}{3}$ for an angle of $\frac{2\pi}{3}$. Both graphs are circles with a radius of 2, so $\frac{2\pi}{3}$ is 1/3 of the circumference which gives $\frac{4\pi}{3}$. Choice **A**.

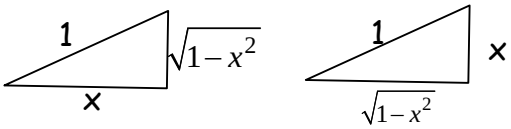
27. Using DeMoivre's theorem, to get 2cis30, 2cis150 and 2cis270 in degrees, which gives -2i, $\pm\sqrt{3}+i$ and $a^2+c^2=5$ which is choice **B**.

28. 

Area is 1/2 times apothem times perimeter or 1/2 times 10/tan22.5 times 160. This gives area of the octagon is 800/z. The circle's radius is 10/sin22.5 which is 10/x. So the area of the requested portion is $100\pi / x^2$ - 800/z. But if we change the last part to 800y/x and get a common denominator, we get choice **B**.

29. $\cos(Arc\cos(x) - Arc\sin(x))$

$= \cos(Arc\cos x)\cos(Arc\sin x) + \sin(Arc\cos x)\sin(Arc\sin x)$



$= x(\sqrt{1-x^2}) + \sqrt{1-x^2}(x) = 2x\sqrt{1-x^2}$ **C**.

30. Square to get $\sqrt{2x+1} = 4 + \sqrt{x-3}$

$2x+1 = 16 + x - 3 + 8\sqrt{x-3}$ or

$x - 12 = 8\sqrt{x-3}$

$x^2 - 24x + 144 = 64(x-3)$ or

$x^2 - 88x + 336 = 0$

(x-4)(x-84)=0 (factor 336 to 3, 4, 4, 7 and to get a large 88 we use 3(4)(7) and 4. The answer x=4 does not give a true equality, so we discard it. The answer x=84 works, and so the square root of 100-x is 4. Answer **D**.