2006 Palm Harbor Invitational Pre-Calculus Individual Test

Mark the best answer on your scantron. NOTA stands for None of the Above. Follow the order of operations. Good luck! Written by Akash P. and Emily L.

1. Simplify: i^i.

A. $e^{\frac{\pi}{2}}$ B. $e^{\frac{-\pi}{3}}$ C. $e^{\frac{-\pi}{2}}$ D. $e^{\frac{\pi}{3}}$ E. NOTA

2. Simplify: $(2+2i)^{11}$

A. $2^{16}i$ B. $2^{16}(1+i)$ C. $2^{16}(i-1)$ D. $2^{16}(1-i)$ E. NOTA

3. What is $|3i + 4|$?

A. 7 B. 1 C. 12 D. 4/3 E. NOTA

4. Which of the following is not a sixth root of 64?

A. 2 B. -2 C. 2 cis (-60°) D. $-1+\sqrt{3i}$ E. NOTA

5. Which of the following is equivalent to $e^{8ix} + e^{-8ix}$?

A. 2cos(8x) B. –sin(8x) C. 2sin(8x) D. –cos(8x) E. NOTA

6. Find the period of $\dfrac{\sin 6x - \sin 4x}{\cos 6x + \cos 4x}$.

A. $\dfrac{\pi}{2}$ B. π C. 2π D. $\dfrac{2\pi}{3}$ E. NOTA

7. What is the amplitude of $y = [27\sin(2x)][52\cos(2x)]$?

A. 140 B. 702 C. 1404 D. 7020 E. NOTA

8. Find the equation in rectangular coordinates: r = 2csc θ + sin θ

A. $x^3 + xy^2 - 3x^2 - 2y^2 = 0$ C. $2x^3 - 3x^2 + 3x^2y + y^2 = 0$ E. NOTA
B. $-y^3 + 3y^2 - x^2y + 2x^2 = 0$ D. $2x^3 + 3xy^2 + x^2 - 3y^2 = 0$

9. Given the equation r = 1+cos θ, how many petals are in the polar graph?

A. 3 B. 2 C. 6 D. 12 E. NOTA

10. Give the particular equation of the conic section described: a locus of points equidistant from a directrix x = 5 and a focus at ordinate = 2 and abscissa = -1.
A. $y^2 + 12x - 4y - 20 = 0$ C. $-x^2 + 3y + 4x - 10 = 0$ E. NOTA
B. $y^2 + 3x - 4y - 2 = 0$ D. $-x^2 + 12y + 4x - 28 = 0$

11. Transform the polar graph of $r=\cos(\theta)$ into a Cartesian equation given $0 \leq \theta \leq 2\pi$

A. $x^2 + y^2 - x = 0$ B. $x^2 + y^2 - y = .2$ C. $x^2 + y^2 - x = .2$ D. $x^2 + y^2 - y = 0$ E. NOTA

12. What is the angle of rotation needed to eliminate the xy-term from $3x^2 + 9xy - 8y^2 + 6x + 5y + 4 = 0$? Round to the nearest tenth of a degree.

A. 60.9° B. 30.6° C. 39.3° D. 19.6° E. NOTA

13. What is the angle between the lines $14x - 5y = -7$ and $4x + 8y = 7$? Round to the nearest degree.

A. 36 B. 126 C. 66 D. 96 E. NOTA

14. What is the area of a triangle with sides measuring 3 cm, 6 cm, and 5 cm?

A. $2\sqrt{7}$ B. $2\sqrt{14}$ C. $\sqrt{70}$ D. $\sqrt{21}$ E. NOTA

15. $\lim_{x\to 0} \left(\frac{\sec 4x - 1}{x}\right)$

A. -1 B. 4 C. 0 D. Does not exist E. NOTA

16. If the letters of COCONUT were placed on a keyring, how many permutations would be possible?

A. 90 B. 840 C. 1260 D. 5040 E. NOTA

17. Molly sold 72 frogs to Anne for \$q22.5z. Find q + z if $q \neq 0$.

A. 5 B. 7 C. 8 D. 9 E. NOTA

18. The King of All Cosmos was rolling his katamari during the winter, lost control, and dropped it in the ocean. After the ocean froze over, he removed the katamari, leaving a hole 16 meters across and 4 meters deep. What was the diameter of the katamari, assuming that it was a perfect sphere?

A. 8 B. 10 C. 20 D. 24 E. NOTA

19. How many zeroes are at the end of (2006!)?

A. 498 B. 499 C. 500 D. 501 E. NOTA

20. Find the sum of #s in the second column in the inverse of the given matrix:

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{bmatrix}$$

A. 2 B. 14 C. 29 D. Not invertible E. NOTA

21. Convert $.\overline{2}_5$ to base 10.

A. $\frac{2}{9}$ B. $\frac{4}{9}$ C. $\frac{2}{5}$ D. $\frac{1}{2}$ E. NOTA

22. At a point 2.5 feet from the center of a merry-go-round, the linear speed is 4 $\text{ft}\cdot\text{s}^{-1}$. How quickly, in $\text{meters}\cdot\text{s}^{-1}$, will a point go if it is 10 feet from the center? Round to the nearest thousandth. (1 foot = .3048 meters)

A. 4.877 B. 5.568 C. 16.000 D. 52.493 E. NOTA

23. Jimmy makes a deposit of $12,500 at the local bank with an interest rate of 3.25% (compounded monthly), forgets about it for a period of time, and later discovers that he now has $16,712.50. How many years has it been since his deposit? Please round to the nearest thousandth of a year.

A. 8.935 B. 8.936 C. 8.948 D. 10.369 E. NOTA

24. Mr. Darcy deposits £1000 at the beginning of each year into a bank account which earns 5.25% interest compounded annually. How much money is present in his account at the beginning of the 12^{th} year?

A. £15149.41 B. £16149.41 C. £16997.25 D. £17997.25 E. NOTA

25. When Pikachu Park first opened, there were five ducks residing in the pond. Nine years later, there are 314 ducks. Assuming that the number of ducks grows according to an exponential function, how many ducks will there be sixteen years after opening day of Pikachu Park? Round to the nearest duck.

A. 2718 B. 7856 C. 7858 D. 7859 E. NOTA

26. Find the magnitude of the vector projection of $\vec{a}$ on $\vec{b}$, given that a $\vec{a} = 6i+7j+3k$, $\vec{b}$ = $6i-2j+4k$.

A. $\frac{17}{47}(6i+7j+3k)$ B. $\frac{17\sqrt{94}}{47}$ C. $\frac{17\sqrt{14}}{14}$ D. $\frac{17}{32}(6i-2j+4k)$ E. NOTA

27. Find the measure (in degrees) of the smaller dihedral angle between the xy plane and $7x+y-4z+7 = 0$. Round to the nearest tenth of a degree.

A. 15.4° B. 60.5° C. 119.5° D. 190.1° E. NOTA

28. If the shortest distance from the line $5x + 12y + 7 = 0$ and the point $(2, y_1)$ is five, find y_1 if $y_1>0$.

A. 10 B. 7 C. 5 D. 4 E. NOTA

29. If $f(3x+5) = 7x - 9$, what is $f(3) - f(6)$?

A. -7 B. 0 C. 7 D. 19 E. NOTA

30. I.) When Akash does not eat, sleep, or drink, you may be sure he is thinking about math.
II.) Tiffany talks about the meaning of life, like she usually does.
III.) Alice is not playing ultimate Frisbee when Akash, Alice, and Tiffany make this a hard pre-calculus test.
IV.) Akash does not think about math on the days that Tiffany talks about the meaning of life, like she usually does.
V.) Akash eats, sleeps, or drinks only if Alice is playing ultimate Frisbee.

From the 5 statements above, which of the following is a VALID conclusion?

A.) Akash does not eat, sleep, or drink
B.) This is a hard pre-calculus test if Tiffany is talking about the meaning of life, like she usually does.
C.) Alice is not playing ultimate Frisbee.
D.) Akash, Alice, and Tiffany have not made this a hard pre-calculus test.
E.) NOTA

2006 Palm Harbor Invitational Pre-Calculus Individual Solutions

Answers:

1. C	7. B	13. E	19. C	25. D
2. C	8. B	14. B	20. B	26. C
3. E	9. E	15. C	21. D	27. B
4. E	10. A	16. A	22. A	28. D
5. A	11. A	17. D	23. C	29. A
6. B	12. D	18. C	24. B	30. D

2006 Palm Harbor Invitational Pre-Calculus Individual Solutions

1. $i^i = e^{\frac{\pi i \times i}{2}} = e^{\frac{-\pi}{2}}$, **C**

2. $(2+2i)^{11} = 2^{11}(1+i)^{11} = 2^{11}(2i)^5(1+i) = 2^{16}i(1+i) = 2^{16}(i - 1)$, **C**

3. $|3i+4| = \sqrt{3^2 + 4^2} = 5$, **E**

4. All are roots of -64, **E**

5. $(\cos 8x + i\sin 8x) + (\cos 8x - i\sin 8x) = 2\cos(8x)$, **A**

6. $\dfrac{2\cos(5x)\sin x}{2\cos(5x)\cos x} = \dfrac{\sin x}{\cos x} = \tan x$ → period = π, **B**

7. Amplitude = $\dfrac{ab}{2} = \dfrac{27(52)}{2} = 702$, **B**

8. $r = \sqrt{x^2 + y^2}$, $x = r\cos\Theta$, $y = r\sin\Theta$

$\sqrt{x^2 + y^2} = 2\dfrac{r}{y} + \dfrac{y}{r}$

$x^2 + y^2 = 4\dfrac{r^2}{y^2} + 4 + \dfrac{y^2}{r^2}$

$-y^3 + 3y^2 - x^2y + 2x^2 = 0$, **B**

9. The graph represented is a cardiod which has no petals. **E**

10. The conic described is a parabola, p=-3 → vertex (2,2)
$x - 2 = -\frac{1}{12}(y - 2)^2$ → $y^2 + 12x - 4y - 20 = 0$, **A**

11. $r = \sqrt{x^2 + y^2}$, $x = r\cos\Theta$
$r^2 = r\cos\Theta$
$x^2 + y^2 = x$
$x^2 + y^2 - x = 0$, **A**

12. A=3, B=9, C=-8
$.5\tan^{-1}(9/11) = 19.6°$, **D**

13. $14x - 5y = -7$ → slope $m_2 = 2.8$ and $4x + 8y = 7$ → $m_1 = -2$
Let A be the angle between the two lines. tan A = (-2 – 2.8)/(1 + -2*2.8) → A = 46°, **E**

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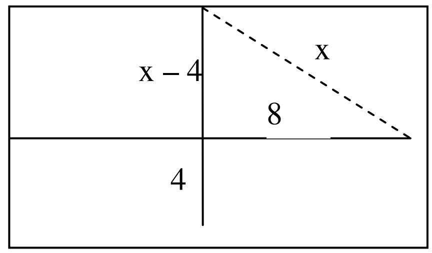
14. s = (3+6+5)/2 = 7
$\sqrt{7(4)(1)(2)} = 2\sqrt{14}$, **B**

15. **[#15 solutions], C**

16. (6!)÷2÷4 = 90, **A**

17. $72 = 3^2 * 2^3$. z must be an even number , and 25z must be divisible by 8. Therefore z = 6.
q+2+2+5+6 = q + 15 must be divisible by 9, q = 3. 6+3 = 9, **D**

18. Cross-sectional view of katamari; figure not drawn to scale.
radius = x
$8^2 + (x-4)^2 = x^2 \rightarrow$ radius = 10, diameter = 20; **C**



19. Let [|x|] denote the greatest integer function.
[|2006/5|] + [|2006/25|] + [|2006/125|] + [|2006/625|] = 500, **C**

20. $$\begin{bmatrix} \begin{vmatrix} 7 & -10 \\ -7 & -15 \end{vmatrix} & -\begin{vmatrix} 3 & -10 \\ -5 & -15 \end{vmatrix} & \begin{vmatrix} 3 & 7 \\ -5 & -7 \end{vmatrix} \\ -\begin{vmatrix} 2 & -1 \\ -7 & -15 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ -5 & -15 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ -5 & -7 \end{vmatrix} \\ \begin{vmatrix} 2 & -1 \\ 7 & -10 \end{vmatrix} & -\begin{vmatrix} 1 & -1 \\ 3 & -10 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -175 & 95 & 14 \\ 37 & -20 & -3 \\ -13 & 7 & 1 \end{bmatrix} = \begin{bmatrix} -175 & 37 & -13 \\ 95 & -20 & 7 \\ 14 & -3 & 1 \end{bmatrix}$$
= 37 – 20 – 3 = 14, **B**

21. $\frac{2}{5} + \frac{2}{25} + \frac{2}{125} \ldots = \frac{\frac{2}{5}}{1-\frac{1}{5}} = \frac{1}{2}$, **D**

22. $r_1 = 2.5$ ft $\rightarrow C_1 = 2\pi r_1 = 5$ ft; $\text{speed}_1 = 4$ ft/s
$r_2 = 10$ ft $\rightarrow C_2 = 2\pi r_2 = 20$ ft; $\text{speed}_2 = x$
$\frac{5\pi}{4} = \frac{20\pi}{x}$; x = 16 ft/s
$\frac{16ft}{1s} \times \frac{.3048m}{1ft}$ = 4.877 m/s, **A**

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23. $16712.50 = 12500(1+\frac{.0325}{12})^{12t}$

$1.337 = (1+\frac{.0325}{12})^{12t}$

$\frac{\ln 1.337}{\ln(1+\frac{.0325}{12})} = 12t \rightarrow t = 8.948$, **C**

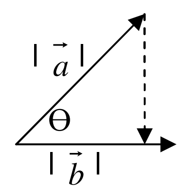
24. $1000(1+.0525)^{11}+\ldots+1000(1+.0525)^{0} = \frac{1000(1-1.0525^{12})}{1-1.0525} = 16149.41$, **B**

25. $P(0) = 5 = a \times b^0 \rightarrow a = 5$

$P(9) = 5 \times b^9 = 314 \rightarrow b \approx 1.5840661$

$P(16) = 5 \times 1.5840661^{16} = 7858.6 \rightarrow 7859$, **D**

26. $|\vec{a}|\cos\Theta = \frac{(\vec{a})(\vec{b})}{|\vec{b}|} = \frac{36-14+12}{\sqrt{36+4+16}} = \frac{17\sqrt{14}}{14}$, **C**



27. $\cos\Theta = \frac{0\times7+0\times1-1\times4}{\sqrt{7^2+1^2+4^2}\times\sqrt{1^2}} \rightarrow \Theta = 119.5°$; $180° - 119.5° = 60.5°$, **B**

28. $5 = \frac{|5(2)+12y+7|}{\sqrt{25+144}} \rightarrow 12y = 48$, $y = 4$, **D**

29. $3x_1+5=3 \rightarrow x_1= -\frac{2}{3}$

$3x_2+5=6 \rightarrow x_2=\frac{1}{3}$

$[7(-\frac{2}{3}) - 9] - [7(\frac{1}{3}) - 9] = -7$, **A**

30. Akash doesn’t eat/sleep/drink → Akash thinks about math
Tiffany talks about the meaning of life.
Akash, Alice, & Tiffany make this a hard test→Alice doesn’t play Frisbee
If Tiffany talks about the meaning of life, Akash doesn’t think about math.
Akash eats, sleeps, or drinks → Alice plays Frisbee.
Akash, Alice, and Tiffany have not made this a hard pre-calculus test, D

2006 PHUHS January Invitational Precalculus Team Question 1

$A = \lim_{x \to 3^-} \frac{|x-3|}{x-3}$

$C = \lim_{x \to 9} \sqrt{x^2 - 49}$

$B = \lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

$D = \lim_{x \to \infty} \frac{2x^2 + 3x + 4}{4x^2 + 1}$

Find:
$A \cdot B \cdot C \cdot D$

2006 PHUHS January Invitational Precalculus Team Question 2

$$\frac{3x^2 + 4x + 4}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$$

Find:
$\frac{A + C - B}{2}$

2006 PHUHS January Invitational Precalculus Team Question 3

A = the measure of the positive angle between the lines $2x-4y=6$ and $5x-6y=12$

B = maximum value of the function $f(x)=4\sin x+32\cos x$

C = the period of $f(x)=2\cos x+3\sin x$

Find to the nearest tenths:

$$\frac{(-A\cdot C)}{4}+B$$

2006 PHUHS January Invitational Precalculus Team Question 4

A = the degree measure of the angle between the vectors $23\vec{i}+92\vec{j}$ and $42\vec{i}+21\vec{j}$, to the nearest hundredth

B = the magnitude of the cross product of the vectors $11\vec{i}+2\vec{j}+21\vec{k}$ and $9\vec{i}+4\vec{j}+3\vec{k}$, to the nearest hundredth

Find, to the nearest hundredth:

$$\frac{-A}{B}$$

2006 PHUHS January Invitational Precalculus Team Question 5

$A = \cos^6 x + 3\sin^2 x \cos^2 x + \sin^6 x$ (hint, simplify)

Given the conic $35x^2 - 92xy + 47y^2 - 24x + 56y - 1 = 0$:

if it is a hyperbola, then $B = -2$,
if it is an ellipse, then $B = 20$,
if it is a parabola, then $B = -200$

Find:

$A - \dfrac{B}{2}$

2006 PHUHS January Invitational Precalculus Team Question 6

Give the function for *y* in terms of *x*, described by the following parametric equations:

$x = \dfrac{1}{\sqrt{t+1}}$, and $y = \dfrac{t}{t+1}$

<u>2006 PHUHS January Invitational Precalculus Team Question 7</u>

$$A = 1 + \cfrac{1}{7 + \cfrac{1}{7 + \cfrac{1}{7 + \cfrac{1}{7 + \dots}}}}$$

$B =$ the real number value of x such that the inverse of the following matrix does not exist:

$$\begin{bmatrix} 2 & 7 & 1 \\ 3 & 1 & 4 \\ 1 & 4 & x \end{bmatrix}$$

C = the distance between the lines $x - 2y = 3$ and $-2x + 4y = 1$

Find the exact value of:

$$\frac{A+B}{C}$$

<u>2006 PHUHS January Invitational Precalculus Team Question 8</u>

Simplify the following expression to a single term:

$$\frac{\cos x + \cos 3x}{\sin 3x - \sin x}$$

2006 PHUHS January Invitational Precalculus Team Question 9

(For this question, you are playing five-card draw poker with a standard 52-card deck, no jokers or wild cards; a straight is five consecutive cards not all the same suit, a flush is five not all consecutive cards of the same suit, and a straight flush is five consecutive cards of the same suit)

A = the probability of making a straight, if you have 5♠ 6♣ 7♥ 8♦ K♦, and discard the K♦ to draw one new card

B = the probability of making a flush, if you have A♠ K♠ 3♠ 2♠ 5♥, and you discard the 5♥ to draw one new card

C = the probability of making a flush, if you have 3♠ 4♠ 5♠ 7♠ Q♦, and you discard the Q♦ to draw one new card

Find:

$$\frac{A+B}{C}$$

2006 PHUHS January Invitational Precalculus Team Question 10

A triangle has perimeter of 60. The lengths of two of the sides sum to 30. The length of one of those two sides is equal to the geometric mean of the other two.

Find, to the nearest tenths of a degree, the measure of the angle opposite the longest side.

2006 PHUHS January Invitational Precalculus Team Question 11

Cody returns from his visit home to China with bird flu. Unfortunately for his Mu Alpha Theta teammates, the spread of the virus is modeled by the function:

$f(t) = \dfrac{200}{1 + 199e^{-0.2t}}$, where t represents the number of days since Cody started spreading the virus.

To the nearest whole day, how many days will it take for half of the 200 total club members to fall ill?

2006 PHUHS January Invitational Precalculus Team Question 12

Find the polar equation of the conic whose eccentricity is 1, whose focus is the origin, and whose directrix is the line $y = 5$.

Solutions:

1) A = -1

B = 4 (l'Hopital's rule)

C = $\sqrt{32} = 4\sqrt{2}$

D = 0.5

ABCD = $-8\sqrt{2}$

2) $3x^2 + 4x + 4 = (A+B)x^2 + Cx + 4A$

A = 1

B = 2

C = 4

(A + C – B) / 2 = 3/2 or 1.5

3) $\tan\theta = \dfrac{\frac{5}{6} - \frac{1}{2}}{1 + (\frac{5}{6})(\frac{1}{2})} = \dfrac{4}{17}$

A = $\tan^{-1}\theta = 13.2$

B = $\sqrt{4^2 + 32^2} = 4\sqrt{65}$

C = 2π

$\dfrac{(-A \cdot C)}{4} + B = 11.5$

4) $2898 = \left(\sqrt{8993}\right)\left(\sqrt{2205}\right)\cos\theta$

$A = \theta = 49.40$

$$B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 11 & 2 & 21 \\ 9 & 4 & 3 \end{vmatrix} = \left| -78\vec{i} + 156\vec{j} + 26\vec{k} \right| = 176.34$$

$\dfrac{-A}{B} = -0.28$

5) A = 1

B = -200, since $B^2 - 4AC = 1884 > 0$, meaning it is a hyperbola

$A - \dfrac{B}{2} = 101$

6) $x^2 = \dfrac{1}{t+1}$

$x^2 + y = \dfrac{1+t}{t+1} = 1$

$y = 1 - x^2$

7) To solve A, let $x = \cfrac{1}{7+\cfrac{1}{7+\cfrac{1}{7+\cfrac{1}{7+\dots}}}} = \dfrac{1}{7+x}$.

$x^2 + 7x - 1 = 0$, and since the positive solution is required, $x = \dfrac{-7+\sqrt{53}}{2}$, so $A = x + 1 = \dfrac{-5+\sqrt{53}}{2}$.

To solve B, the inverse will not exist if the determinant is equal to 0. Solving by minors using the bottom row:

$(28-1) - 4(8-3) + x(2-21) = 0$, and $B = x = \dfrac{7}{19}$.

To solve C, use the distance formula for a point to a line (since the two lines are parallel). A convenient point for the second line is $(0, 0.25)$, so:

$C = \dfrac{|1(0) - 2(0.25) - 3|}{\sqrt{1^2 + (-2)^2}} = \dfrac{3.5}{\sqrt{5}} = \dfrac{7\sqrt{5}}{10}$.

Thus, $\dfrac{A+B}{C} = \dfrac{\dfrac{-5+\sqrt{53}}{2} + \dfrac{7}{19}}{\dfrac{7\sqrt{5}}{10}} = \left(\dfrac{-81+19\sqrt{53}}{38}\right)\left(\dfrac{\sqrt{5}}{3.5}\right) = \dfrac{-81\sqrt{5}+19\sqrt{265}}{133}$.

8) $\dfrac{\cos x + \cos 3x}{\sin 3x - \sin x} = \dfrac{2\cos(2x)\cos(-x)}{2\cos(2x)\sin(x)} = \cot x$

9) A = $\dfrac{8}{47}$ (four 4's and four 9's will complete the straight, out of 47 unseen cards left in the deck)

B = $\dfrac{9}{47}$ (nine spades will complete the flush, out of 47 unseen cards left in the deck)

C = $\dfrac{8}{47}$ (eight spades will complete the flush – the 6♠ will make a straight flush, out of 47 unseen cards left in the card)

Thus, $\dfrac{A+B}{C} = \dfrac{\dfrac{8}{47} + \dfrac{9}{47}}{\dfrac{8}{47}} = \left(\dfrac{17}{47}\right)\left(\dfrac{47}{8}\right) = \dfrac{17}{8}$

10) Let the three sides be x, y, and z. We are given, then:

$x + y + z = 60$

$x + y = 35$ and $z = 25$

$x = \sqrt{yz}$.

First, solve for the side lengths:

$x = \sqrt{25y} = 35 - y$

$y^2 - 95y + 1225 = 0$

$y = \dfrac{95 - 5\sqrt{165}}{2} \approx 15.387$ (the other solution is extraneous, greater than 35)

$x = 35 - \dfrac{95 - 5\sqrt{165}}{2} = \dfrac{-35 + 5\sqrt{165}}{2} \approx 19.613$.

Use the law of cosines to solve for the angle opposite the longest side z:

$25^2 = 15.387^2 + 19.613^2 - 2(15.387)(19.613)(\cos x)$

$x = \cos^{-1}(-0.0059) \approx 90.3$

11) $100 = \dfrac{200}{1 + 199^{-0.2t}}$

$t = \dfrac{\ln \frac{1}{199}}{-0.2} \approx 26$

12) $r = \dfrac{ep}{1 + e\sin\theta} = \dfrac{5}{1 + \sin\theta}$

13) For part A, find the numbers in the range with 1 or 8 factors (27 is too high). 1 is the only number with 1 factor. Of the 8 factor numbers, there are 5 numbers with 2 prime factors (24, 40, 54, 56, 88), there are 4 numbers with 3 prime factors (30, 42, 56, 88). Thus, A = 10.
For part B, there are 27 prime numbers with 2 factors, 4 numbers with 3 factors (4, 9, 25, 49), 2 numbers with 5 factors (16, 81), 1 number with 7 factors (64). Thus, B = 34.
B (mod A) = 34 (mod 10) = 4

14) To solve, use the following table:

	Silence	Confession
Silence	3, 3	9, 1
Confession	1, 9	6, 6

The first column are your choices, and top row are your accomplice's choices. It is obvious that not matter what your accomplice chooses to do, it is to your advantage to confess (if he remains silent, you get 1 year instead of 3; if he confesses, you get 6 instead of 9).

Thus, for A, your expected value is $(0.5)(1)+(0.5)(6)=3.5$.
For B, since your accomplice is as smart as you are, he will decide to confess as well, meaning that you will get 6 years.
Finally, $A \cdot B = 21$.

15) For your second and final choice, you only have a one-third chance of picking the million dollar door if you stick to your original choice. Sticking with your choice also means you have a two-thirds chance of getting the 90% door. If you switch your choice to the so far unnamed door, you have a two-thirds chance of winning the million (because of the extra given information, which collapses all the probabilities that you would not have picked the million-dollar door to the one that should be switched to), and a one-third chance of getting the 90% door. Switching your door gives the higher expected value, which is:
$\frac{2}{3}(1,000,000)+\frac{1}{3}(.9)(500,00)=816,666.67$ dollars.