Palm Harbor University High – Mu Alpha Theta
2006 – January Invitational – Geometry Individual Test

For all questions, E. NOTA means "none of the above" answers are correct.

1. In rhombus ABCD, AB = 6 and BD = 8. What is the area of rhombus ABCD?

A. $16\sqrt{5}$ B. 24 C. $24\sqrt{5}$ D. 48 E. NOTA

2. In triangle ABC, the angle bisector of <A intersects side BC at point D. If BD = 8, AB = 16 and the perimeter of triangle ABC is 42, then what is the length of segment AC?

A. 12 B. 16 C. 16 D. 24 E. NOTA

3. Ryan, an architect, designed a rather peculiar building. The main part of the building was a rectangular prism with square bases, however, on top of the building was a semi-spherical dome which was tangent to all sides of the top of the building. If the height of the dome is equal to half of the height of the rest of the building, and the area of the base is 144 square yards, how tall is the entire building (in feet)?

A. 18 B. 36 C. 48 D. 54 E. NOTA

4. The sum of the interior angles of a regular polygon is 5040 degrees. How many diagonals does this polygon have?

A. 168 B. 360 C. 405 D. 810 E. NOTA

5. Taryn lives in A town. She wants to visit her friend Erin in D town. If B town is 60 miles due south of A town, C town is 84 miles due west of B town, and D town is 348 miles due north of C town, then to the nearest mile, how far would Taryn have to drive from A town to D town, assuming she likes to go off-roading and travels in a straight line between the two?

A. 144 B. 288 C. 300 D. 324 E. NOTA

6. Which of the following is a possible coordinate of the center of a circle with a radius of $\sqrt{113}$ and a tangent line at $8(y -3) = 7(x +1)$?

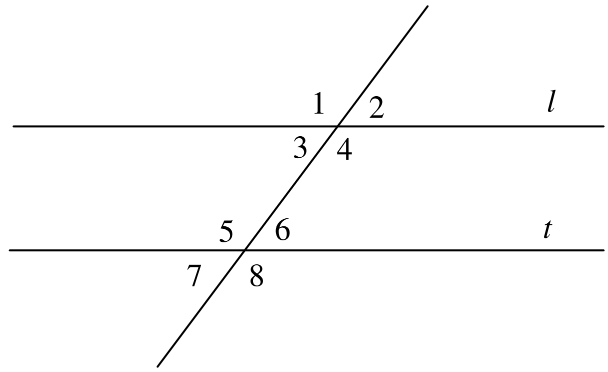
A. (15,17) B. (-8,11) C. (-1,3) D. (0,0) E. NOTA

7. In the given figure with line *l* and line *t*, which of the following must be true?

I. $\angle 1 = \angle 4$

II. <2 = <6

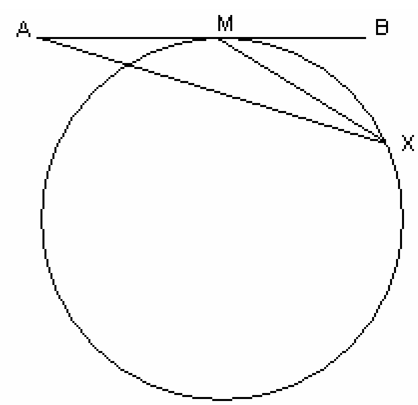
III. m<1 + m<2 = 180



A. I only B. I, II only C. I, II, III only D. I, III only E. NOTA

8. In the figure below, line segment AB is tangent to the given circle at point M. If segment MX is a chord of the circle, then what is the measure of $\angle AMX$, given that minor arc $\overset{\frown}{MX}$ has a measure of $58°$?

A. 116 B. 122 C. 141 D. 151 E. NOTA



9. A trapezoid is inscribed in a circle with radius 12. One of the parallel sides of the trapezoid is a diameter of the circle, while the other has length 18. What is the height of the trapezoid?

A. $3\sqrt{7}$ B. $9\sqrt{7}$ C. $18\sqrt{7}$ D. $27\sqrt{7}$ E. NOTA

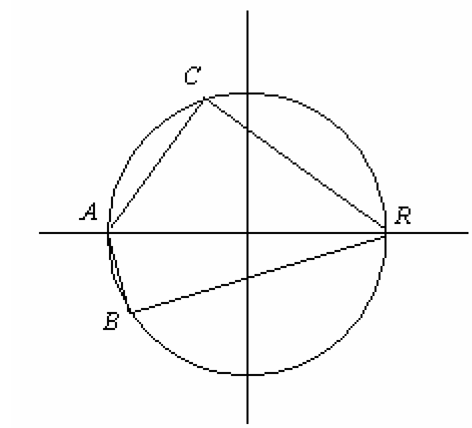
10. The length of a side of an equilateral triangle is 18. What is the area of the circumscribed circle?

A. 72π B. 84π C. 96π D. 108π E. NOTA

11. Find the area of a regular octagon with a perimeter of 96 cm?

A. 576 cm^2 B. 324π cm^2 C. 1008 cm^2 D. 1256 cm^2 E. NOTA

12. In the figure at right, if $AB = \frac{1}{5}AC$, CR = 24, and AB = 2, then to the nearest tenth, what is the area of quadrilateral ACRB?



A. 120.0 B. 132.7 C. 145.9
D. 155.7 E. NOTA

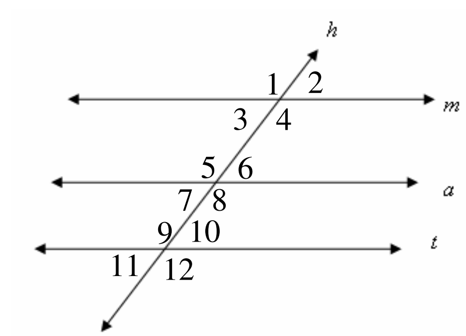
13. Farmer Akash wants to fence-in a pond in his backyard. The pond just happens to be a perfect circle. If Farmer Akash can only buy fencing in the shape of a regular hexagon, what is the perimeter(p) of the smallest fence that Farmer Akash can buy that will completely enclose the pond, which has an area of 64π?

A. $30 < p \leq 40$ B. $40 < p \leq 50$ C. $50 < p \leq 60$ D. $60 < p \leq 70$ E. NOTA

14.If Rikin needs to bring 10 soccer balls for his team's practice and a size 5 soccer ball has a circumference of 20 π inches. How much truck space would Rikin need if he could stack 1 ft^3 of balls in 1.5 cubic feet of truck space? Round to the nearest tenth of a cubic foot.

A. 4.5 ft^3 B. 13.7 ft^3 C. 36.4 ft^3 D. 7850.0 ft^3 E. NOTA

15. I. <1 = <4
II. <1 = <5
III. <6 + <8 = 180
IV. <8 - <9 = 0
V. <8 + <12 = 180



Which of the above MUST be true for *m* || *a* || *t*?

A. I, III, & IV B. II, III, & V C. II, & IV
D. All of the above. E. NOTA

16. If Cowboy Dan is building a fence for his cattle along a river. If he has 400 yards of fence, what is the largest area of enclosed rectangular pasture he can make? Assume the cattle won't swim and are "enclosed" by the water.

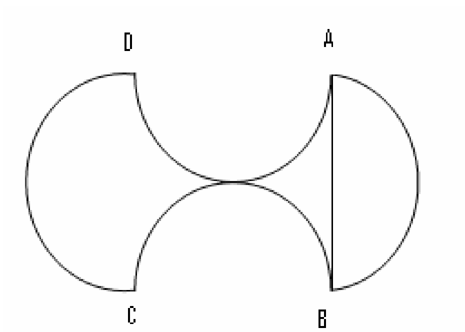
A. 20,000 yd^2 B. 40,000/π yd^2 C. 16,000/9 yd^2 D. 16,000 yd^2 E. NOTA

18. If side *a* >side *b*, and sin A > sin C. Which of the following inequalities will <u>always be true</u>?

A. $c > a > b$ B. $a > c > b$ C. $a > b > c$ D. $c > a - b$ E. NOTA

19. At what point does the angle bisector of BUC intercept the segment BC if B (10,14), U (-5,35), and (70,42)?

A. (47,32) B. (40,28) C. (52,26) D. (55,36) E. NOTA



Use this figure for Problems 20 and 21.

20. If Quadralateral ABCD is a square with a side of 3, what is the area of this shape?

A. 3π B. 2.25 π C. 9 D. 6 √ 2 E. NOTA

21. What is the perimeter of this figure?

A. 1.5 π B. 3π C. 6 π D. 12 π E. NOTA

22. What is the closest distance between y = ¾ x + 6 and 3x - 4y = 21?

A. 7.20 B. 9.00 C. 15.00 D. 25.00 E. NOTA

23. The amount of ice cream is directly proportional to the volume of the ice cream cone. How much ice cream would a large cone hold if it was twice as high and three times as wide as the small ice cream cone which can hold 6 ounces?

A. 36 oz B. 54 oz. C. 108 oz D. 216 oz E. NOTA

24. What is the height of a regular pyramid with 4 vertices and a base with an area of $36\sqrt{3}$?

A. 3 B. $4\sqrt{6}$ C. $6\sqrt{3}$ D. $9\sqrt{3}$ E. NOTA

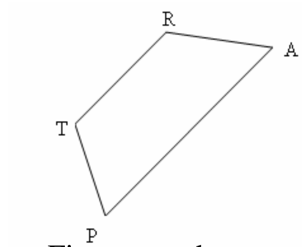
25. Which of the following are not possible lengths of a diagonal of a quadrilateral with all the sides with a length of 6?

A. 2 B. 6 C. 10 D. 12 E. NOTA

26. Given lines TR and AP are parallel and angles A and P are supplementary. Which of the following MUST be true?

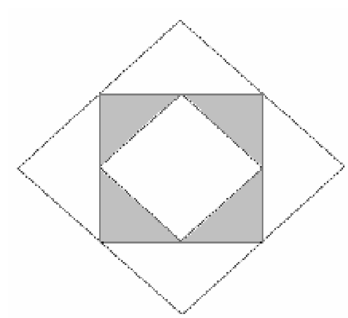
A. Angles T and P are equal B. Angles R and P are supplementary

C. TP = RA D. All of the Above E. NOTA



Note: Figure not drawn to scale.

27. In the figure to the right, each of the "nested" quadrilaterals' vertices are at the midpoints of the sides of the quadrilateral within which it is nested. If the area of the shaded region is 9, what is the perimeter of the outer quadrilateral?



A. $24\sqrt{2}$ B. 24 C. $12\sqrt{2}$ D. 12 E. NOTA

28. What is the arc length formed by an angle with a measure of 145° with a radius of 8?

A. $\frac{29\pi}{72}$ B. $\frac{29\pi}{9}$ C. $\frac{58\pi}{9}$ D. $\frac{116\pi}{9}$ E. NOTA

29. Let BRSU be a rhombus with the shorter diagonal length of 18 and the longer diangonal with a length of $32 - 2x^2$. What is a possible value of x be if SU = 15?

A. 2 B. 4 C. 6 D. 12 E. NOTA

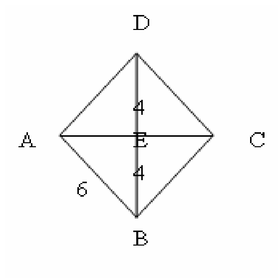
30. Vibha and Channing are cooking muffins for a birthday party. They have enough mix to make 0.01 cubic foot of muffins, which is just enough for all 27 students in their math class. How tall will the cylindrical muffins be if they have a base area of 0.80 in^2?

A. 0.8 inches B. 1.0 inch C. 1.2 inches D. 2.0 inches E. NOTA

1. Since polygon ABCD is a rhombus, its diagonals bisect each other so BE = DE = 4.
Using Pythagorean Theorem:

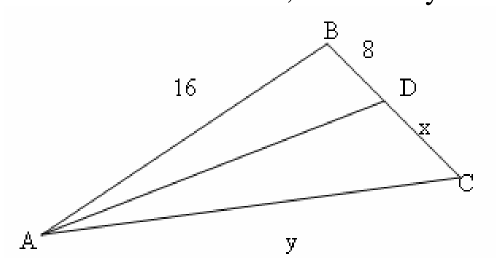
$AE^2 + 4^2 = 6$
$AE^2 = 36 - 16 = 20$
$AE = \sqrt{20} = 2\sqrt{5}$

Area of rhombus = 4 (Area of triangle ABE)
Area = $4\,(1/2)(4)(2\sqrt{5})$ = **$16\sqrt{5}$, A**



2. Since angles BAD and CAD are congruent $\frac{BD}{AB} = \frac{CD}{AC} \rightarrow 8/16 = x/y \rightarrow 2x = y$

The perimeter is $42 = 16 + 8 + x + y$
$18 = x + y$
$18 = x + 2x = 3x$
$x = 6$, so $AC = y = 2x = 2 \cdot 6 =$ **12, A**



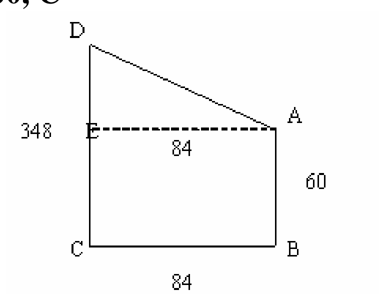
3. If the base is a square with an area of 144 yd^2, its sides are 12 yd. The diameter of the hemisphere must be 12 yd, so the radius is 6 yd. Since the height of the dome is its radius (6yd), the height of the rest of the building is $2 \cdot 6$ yd = 12 yd. The total height of the building is 18 yards or **54 feet, D.**

4. Sum of all interior angles of a polygon with n sides= $180(n - 2)$

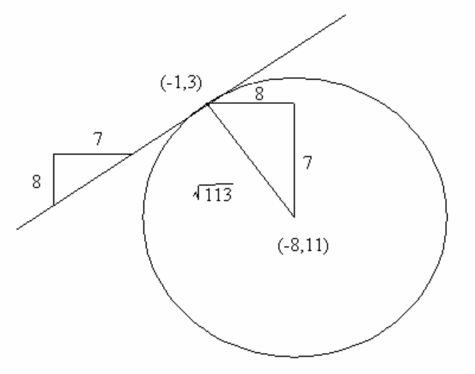
$5040 = 180(n - 2)$
$28 = n - 2$

Since there are 30 sides, the number of diagonals equals
$28 + 27 + 26 + 25 + 24 + \ldots + 4 + 3 + 2 =$ **405, C**

5. $AD^2 = AE^2 + DE^2$
$AD^2 = 84^2 + (348 - 60)^2 = 84^2 + 288^2$
$AD^2 = 90000mi^2 \rightarrow$ **AD = 300, C**



6. $7(y-3) = 8(x + 1)$
$7y - 21 = 8x + 8$
$7y = 8x + 29$
$y = 8/7\ x + 29/7$ poss. tangent point at (-1,3)

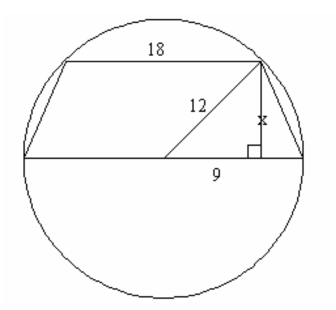


(-8,11) is a possible center, B

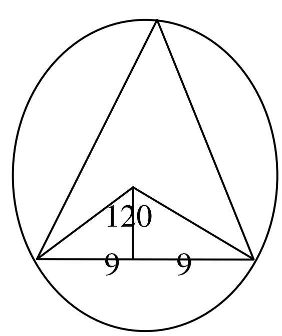
7. I is true, definition of vertical angles,
II is false, lines *l* and *t* are not given as parallel
III is true, definition of supplementary angles
I, III only, D

8. Since the minor arc of MX is 58º is major arc is 302º. Since AB is a tangent at M, its arc is half of the major arc or **151º, D.**

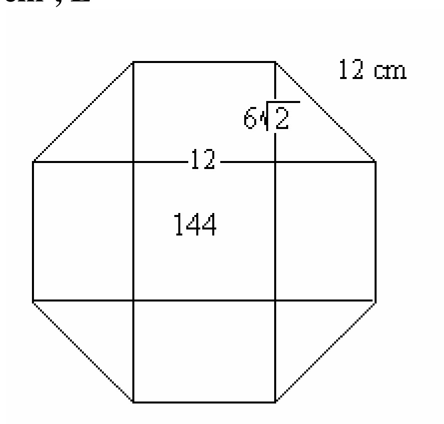
9. $x^2 + 9^2 = 12^2$
$x^2 = 144 - 81 = 63$
$x = \sqrt{63} = \mathbf{3\sqrt{7}}$**, A**



10. By drawing the triangle in the circle, you find a 30-60-90 triangle. The radius is $6\sqrt{3}$.
Area = $\pi\ (6\sqrt{3})^2$ = **108 π, D**



11. The perimeter of a regular octagon is 96 cm therefore the length of the side is 12 cm.
Area = $12^2 + 4 \cdot (6\sqrt{2})(12) + 4 \cdot (1/2)\ (6\sqrt{2})^2$
Area = $288 + 288\sqrt{2} \approx$ **695.3 cm^2, E**

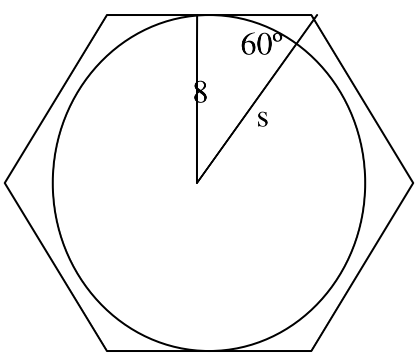


12. AC = 10 because AC/5 = AB = 2. Let angle B and C be a right angle. $AR^2 = CR^2 + AC^2$

$AR^2 = 10^2 + 24^2 = 676$
$AR = 26$
$AR^2 = BR^2 + AB^2$
$26^2 = BR^2 + 2^2$
$676 - 4 = BR^2 = 672 \rightarrow BR \approx 25.92$

Area= ½ (AB)(BR) + ½ (AC)(CR)
= ½ (2 · 25.92) + ½ (10 · 24) = **145.9 , C**

13. Area of pond = $64\pi = \pi r^2 \rightarrow r = 8$
$s = 2 \cdot (8\sqrt{3})/3 = (16\sqrt{3})/3$
Perimeter = $6s = 6 \cdot (16\sqrt{3})/3 = 32\sqrt{3} \approx 55.4$, **$50 < p \leq 60$, C**



14. $2\pi r = 20\pi$ in
r = 10 in
r = 5/6 ft

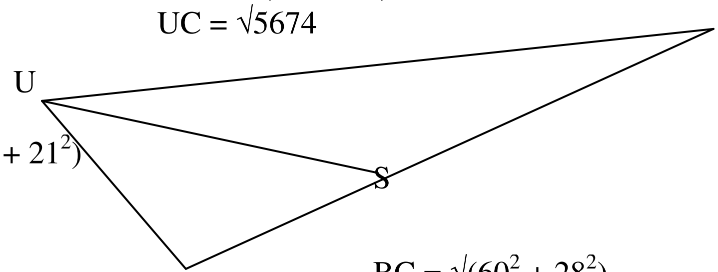
$V = \dfrac{(3/2)(4\pi/3)(5^3)(10)}{6^3} \approx$ **36.4 ft^3, D**

15. I is always true because they're vertical angles.
II must be true to prove *m* and *a* are parallel by corresponding angles.
III is always true because they're supplementary angles.
IV must be true to prove *a* and *t* are parallel.
V is not always true.
II and IV only, C

16. The most effective way of enclosing area when you don't have to fence one side is to make a rectangle with a sides of length x, 2x, and x. Since Dan has 400 yards of fence, x = 100 yards. 200 yd · 100 yd = **20,000 yd^2 of pasture, A**
17. C
18. c > a – b is always true because the sum of two sides of a triangle must be greater than the third, **D.**

19. Graph points and determine the lengths of BU, BC, and UC.

$UC = \sqrt{(75^2 + 72)}$
$UC = \sqrt{5674}$



$UB = \sqrt{(15^2 + 21^2)}$
$UB = \sqrt{666}$

$BC = \sqrt{(60^2 + 28^2)}$
$BC = \sqrt{4384}$

$$\frac{BS}{BU} = \frac{CS}{UC} \rightarrow \frac{xBC}{BU} = \frac{(1-x)\,BC}{UC} \rightarrow (1-x)BC\cdot BU = x\,BC\cdot UC \rightarrow (1-x)\,BU = xUC$$

$(1-x)\sqrt{666} = x\sqrt{5674}$
$\sqrt{666} = x(\sqrt{(5674)} - \sqrt{(666)})$
$x = (\sqrt{666})/((\sqrt{(5674)} - \sqrt{(666)}))$
Point = B + xBC
$([10 + (60)\cdot(\sqrt{666})/((\sqrt{(5674)} - \sqrt{(666)}))], [14 + (28)\cdot(\sqrt{666})/((\sqrt{(5674)} - \sqrt{(666)}))])$
(41.27, 28.59), E

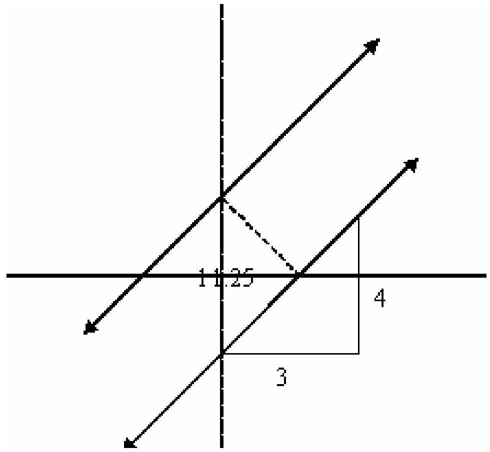
20. You can cut off the two semicircles at AB and CD and put them in the missing spots to form a square with length of 3. So area = 3^2 = **9, C**

21. This figure has the same perimeter as two circles with a diameter of 3. P = 2 · 3π = **6 π, C**

22. $y = \frac{3}{4}x + 6$

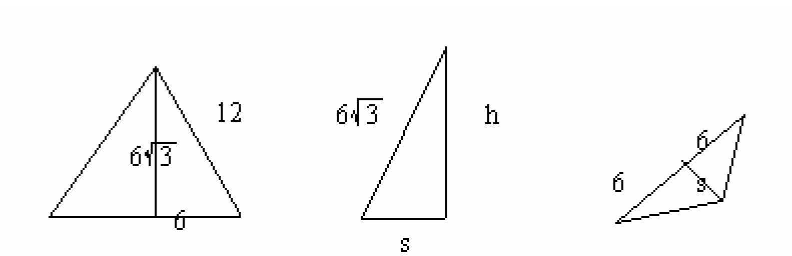
$3x - 4y = 21$
$3x - 21 = 4y$
$\frac{3}{4}x - 21/4 = y$

The difference between the y axis is 11.25. Since it is the hypotenuse of the line perpendicular to both lines it is a multiple of the 3-4-5 Triangle. The distance between the line corresponds to the 4 leg. Distance = 11.25/5 · 4 = **9, B**



23. Let the ice cream cone have a height 1 and a radius of 1 holds 6 oz.
$V = k\pi/3(1)^2(1) = k\pi/3 = 6$ oz
$V_{large} = k\pi/3(3)^2(2) = 18k\pi/3 = 18(6\text{oz}) =$ **108 oz, C**

24. Since the regular pyramid has four vertices it if formed by 4 identical equilateral triangular faces. The edges have a length of 12 units because of the area. The slant height is $6\sqrt{3}$ which is the hypotenuse of a right triangle perpendicular at the center of the triangle. s must be $2\sqrt{3}$ because of 30-60-90 triangle. Using the Pythagorean Theorem, $h^2 + (2\sqrt{3})^2 = (6\sqrt{3})^2$
$h^2 + 12 = 108$
$h^2 = 96$
$h = \sqrt{96} =$ **$4\sqrt{6}$, B**



25. It is impossible for a diagonal of a rhombus to be 12 by the triangle inequality rule. **D**

26. TP = AP are the only ones necessary to from a trapezoid, the other requirements are not true in every case. **B**

27. In solving this problem, the type of quadrilateral is never distinguished so the answers very depending whether on which type of quadrilateral that is solved for. **E. NOTA**

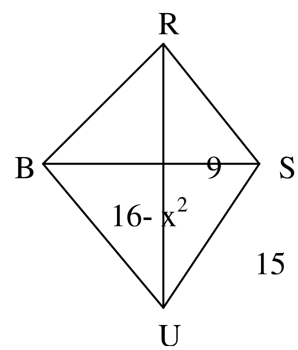
28. arclength = radians · radius
Arclength = 145° (π/180°) (8) = **58π/9, C**

29. When drawing the rhombus, you can see there is a multiple of the 3-4-5 triangle which means ½ (the long diagonal) = 12.

$16 - x^2 = 12$

$4 = x^2$

$x = \pm 2$, **2 is a possible answer, A**



30. $V_{Total} = .01\ ft^3 = .01\ (12)^3 = 17.28\ in^3$

$V_{1\ muffin} = 17.28 / 27 = .64\ in^3$

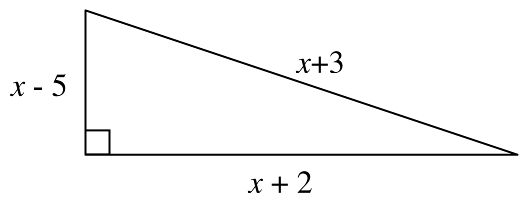
$V = A \cdot h$

$.64\ in^3 = .80 in^2\ h$

h = **.8 in, A**

Palm Harbor University High Invitational – Geometry Team Round Question #1

Solve for x.



Palm Harbor University High Invitational – Geometry Team Round Question #2

Find the surface area of the smallest cube that can inscribe a sphere with a volume of 72π m^3?

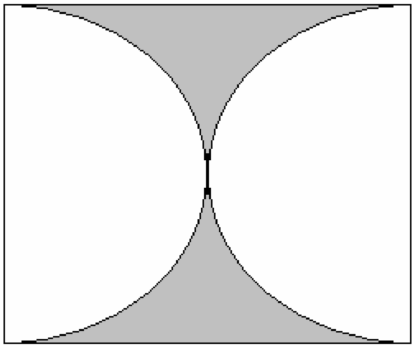
Palm Harbor University High Invitational – Geometry Team Round Question #3

There are four peaks that form a square when you plot the location of their summits on a map. By using the map like a Cartesian coordinate plane, find the locations in the first quadrant of the third and fourth summits if the first and second summits are (-2,6) and (1, -3), respectively.

Palm Harbor University High Invitational – Geometry Team Round Question #4

Fred needs to paint only outsides (not the top or bottom) of 3 dozen identical boxes with a length of 4` 6``, a width of 2`3``, and a height of 1`6``. Given that it takes 1 paint can to cover 12 ft^2, how many whole cans of paint will he need to complete the job.

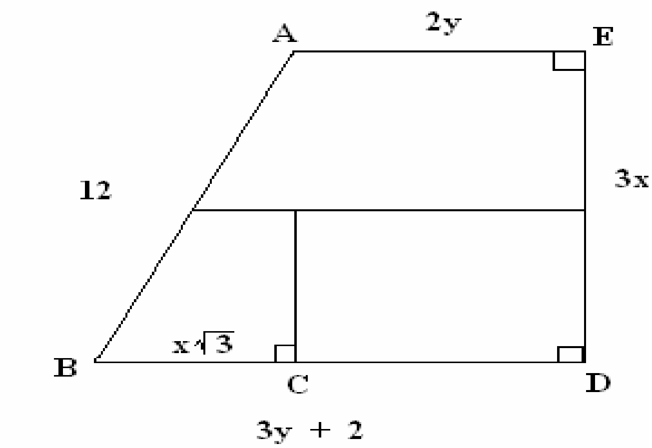
<u>Palm Harbor University High Invitational – Geometry Team Round Question #5</u>



Assuming the unshaded portion of the figure is formed by two semi-circles with diameter equal to the length of the side of the square, what is the ratio of the unshaded portion to the shaded portion?

<u>Palm Harbor University High Invitational – Geometry Team Round Question #6</u>

Find the perimeter of trapezoid ABDE if ACDE is a rectangle.



Palm Harbor University High Invitational – Geometry Team Round Question #7

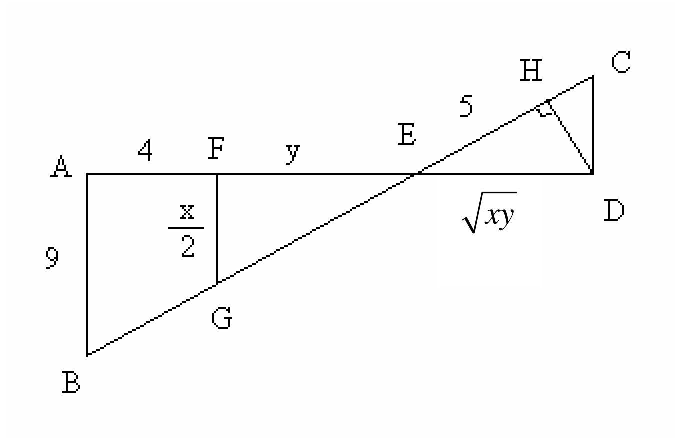
Find the area of a regular dodecagon whose longest diagonal is 12 inches?

Palm Harbor University High Invitational – Geometry Team Round Question #8

If a cube is cut 15 times to form a number of smaller cubes with uniform volumes. What fraction of these new cubes share at least one of their faces with the original cube?

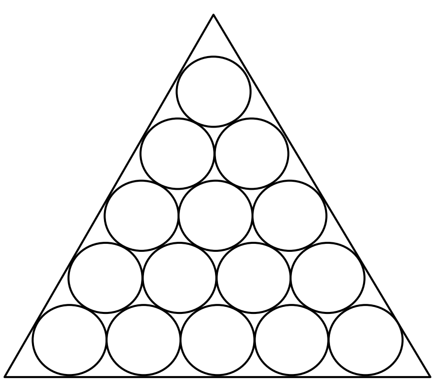
Palm Harbor University High Invitational – Geometry Team Round Question #9

9. Let AB, CD, and FG be perpendicular to AD. What is the value of HD?



Palm Harbor University High Invitational – Geometry Team Round Question #10

10. Find the area of the triangle if all fifteen circles have a radius of 10 units

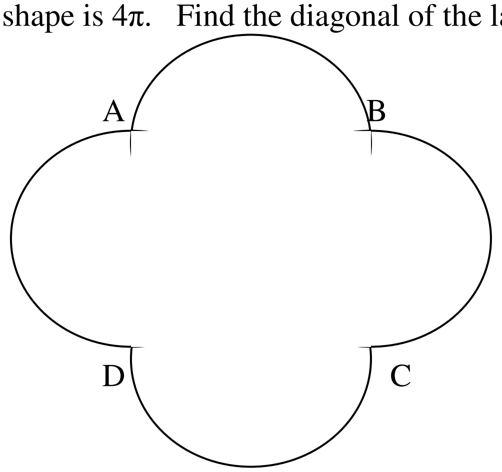


Palm Harbor University High Invitational – Geometry Team Round Question #11

11. If A is the midpoint of BC, D is the midpoint of AE, and BE is bisected by point F. If FD is 10, what is the value of AB?

Palm Harbor University High Invitational – Geometry Team Round Question #12

12. Given: the shape in the figure is composed of four semi-circles. The perimeter of this shape is 4π. Find the diagonal of the largest square that can fit inside this figure.



Palm Harbor University High – Mu Alpha Theta

2006 – January Invitational – Geometry Team Round Solutions

1. Pythagorean Theorem

$(x + 2)^2 + (x - 5)^2 = (x + 3)^2$

$x^2 + 4x + 4 + x^2 - 10x + 25 = x^2 + 6x + 9$

$2x^2 - 6x + 29 = x^2 + 6x + 9$

$x^2 - 12x + 20 = 0$

$(x - 2)(x - 10) = 0$

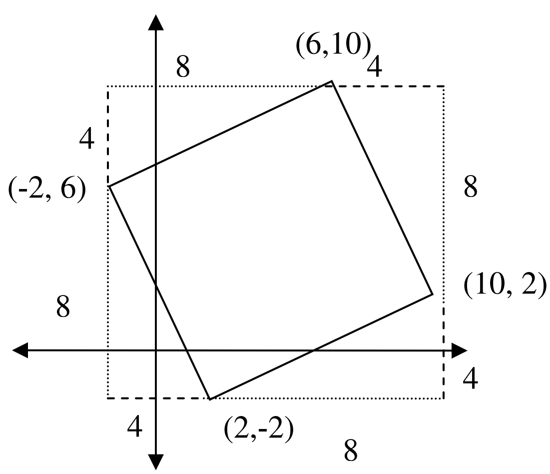
x = 2, 10 but x = 2 is extraneous because a leg can't have a negative length

x = 10

2. A cube with length of the 2r of the sphere

$V = (4\pi/3)\, r^3$	Surface Area = $6\,(2r)^2$
$72\pi\ m^3 = (4\pi/3)\, r^3$	S A = $24\, r^2$
$54\ m^3 = r^3$	S A = $24\, (\sqrt[3]{54})$
$r = 3\sqrt[3]{2}$ m	S A = $24\, (\sqrt[3]{2916}) = 216\sqrt[3]{4}\ m^2$

3.



(6,10) and (10,2) are the two other vertices

4. $3 \cdot 12 = 36$ boxes

Total Area Painted = 36 (2(4.5ft)(1.5ft) + 2(2.25ft)(1.5ft))

Total Area = $729\ ft^2$

$\frac{729\ ft^2}{12\ ft^2} = 60.75$ paint cans → Fred needs 61 paint cans to finish the job

5. Area of Square = s^2

Area of 2 semicircles = $\pi\,(\frac{1}{2}s)^2$

Area of shaded region = $s^2 - \pi\,(\frac{1}{2}s)^2$

Area unshaded : Area shaded

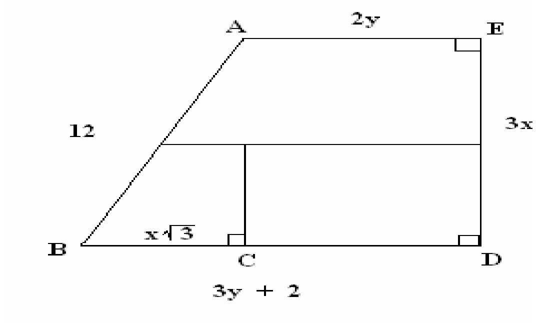
$\pi s^2/4 \ :\ s^2 - \pi s^2/4$

$\pi/4 \ :\ 1 - \pi/4$

$\pi/4 \ :\ 3\pi/4$

$1 : 3$

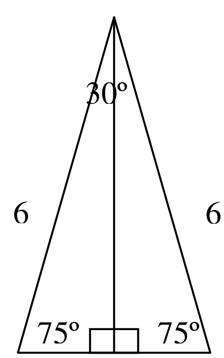
6.



$AB^2 = BC^2 + AC^2$	$BD - BC = AE$
$12^2 = (x\sqrt{3})^2 + (3x)^2$	$3y + 2 - x\sqrt{3} = 2y$
$144 = 3x^2 + 9x^2$	$y = x\sqrt{3} - 2$
$144 = 12x^2$	$y = 2(\sqrt{3})(\sqrt{3}) - 2 = 6 - 2$
$12 = x^2$	$y = 4$

$2\sqrt{3} = x$, ($x = -2\sqrt{3}$ is extraneous lengths are not negative)
Perimeter = $2y + 3x + (3y + 2) + 12$
$P = 5y + 3x + 14$
$P = 5(4) + 3(2\sqrt{3}) + 14 = 34 + 6\sqrt{3}$

7. A dodecagon has 12 sides
If 6 main diagonals are drawn, there are 12 isosceles triangles inside the polygon and the vertex angle of each is 30º (because 360 º/12). The lengths of the identical sides are 6 inches each.



If each triangle is split into two right triangles, the area of each is ½(6sin75º)(6cos75º).
So the area of the dodecagon is ½(6sin75º)(6cos75º)(2)(12)→ Area = 108 in^2

8. 15 cuts means 5 cuts are made on each dimension. There are 6^3 or 216 new cubes. Of the 216 cubes, 4^3 or 64 do not share any side with the original cube. 152 cubes share at least one side with the original cube. Probability = $^{152}/_{216} = {}^{25}/_{27} \approx .9259$

9. Triangles ABE, EFG, and EDH are all similar because all corresponding angles are congruent. Therefore:

$$\frac{AE}{AB} = \frac{FE}{FG} = \frac{HE}{HD} \rightarrow \frac{y+4}{9} = \frac{y}{\frac{1}{2}(x)} \rightarrow 9y = 2x + xy/2$$

$$18y = 4x + xy$$

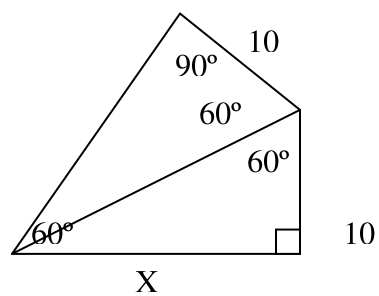
$$18y - xy = 4x$$

$$y(18 - x) = 4x$$

$$y = \frac{4x}{18 - x}$$

By listing possible solution, the students will discover x = 10, y= 5 satisfy the conditions. Therefore, all of the triangles are 45, 45, 90 triangles and HD would equal HE so HD = 5.

10.



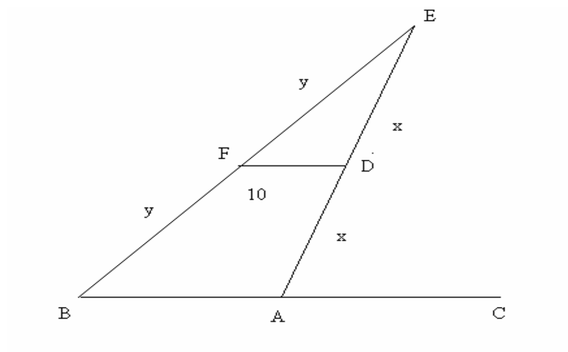
Side X is distance along the side of the triangle between the tangent point and a vertex of the circles in the corner. This length is determined to be $10\sqrt{3}$ because it is a 30-60-90 triangle. The length of the side of the large triangle enclosing 15 circles is $2(10 + 10\sqrt{3}) + 3(20)$. Therefore:

$$s = 80 + 20\sqrt{3}$$

$$\text{Area} = \tfrac{1}{2} s^2 \cos 60^\circ = \frac{((80 + 20\sqrt{3})^2)}{2} \frac{(\sqrt{3})}{2} = (6400 + 3200\sqrt{3} + 1200)\sqrt{3}/4$$

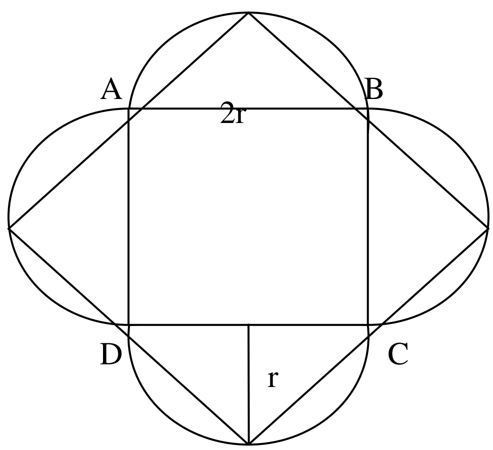
$$A = \frac{(7600 + 3200\sqrt{3})\sqrt{3}}{4} = \frac{9600 + 7600\sqrt{3}}{4} = \quad 2400 = 1900\sqrt{3}$$

11.



$$\frac{AB}{AE} = \frac{DF}{FD} \rightarrow \frac{AB}{2x} = \frac{10}{x} \rightarrow x(AB) = 20x \rightarrow AB = 20$$

12.



Since the perimeter of 4 half circles is 4π, diameter of each of these circles is 2 so $r = 1$. The largest square can fit like shown. The diagonal is $r + 2r + r = 4r$ or $4(1) = 4$ units