January 2006 Palm Harbor University High School Invitational
Algebra I Individual

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Directions: Determine the best answer and mark it on the answer sheet. NOTA means none of these answers. You may use a scientific calculator on this test. Good luck!

1. If $7w = 2x$, $8x = 4y$, $6y = 7z$, and $z = 84$, then what is the sum of w, x, y, and z?

A) 336　　　　B) 182　　　　C) 245　　　　D) 504　　　　E) NOTA

2. Find the slope of the line perpendicular to the line that passes through points (3,7) and (6, 10).

A) 1　　　　B) -3　　　　C) -1　　　　D) 1/2　　　　E) NOTA

3. Find the average (mean) of all roots given by the following equation: $2x^2 - 13x + 21 = 0$

A) 7/2　　　　B) 3　　　　C) 13/2　　　　D) 13/4　　　　E) NOTA

4. Simplify completely:

$$\frac{(2x^{-3}y^4z^{-2})^{-1}}{3x^2y^{-1}z^5}$$

A) $\dfrac{x}{6y^3z^3}$　　　　B) $\dfrac{2y^3}{3z^7x^2}$　　　　C) $6xy^3z^7$　　　　D) $\dfrac{xy}{z}$　　　　E) NOTA

5. Tim wants to improve his shooting average in soccer from 40% to 50% of his shots that he scores. So far this season he has scored 8 goals. If there are 3 games left, to the nearest hundredth, how many goals on average does he need to score PER GAME in order to reach his goal of a 50% average assuming he scores on every shot from here on?

A) 1.00 goal　　B) 2.00 goals　　C) 1.50 goals　　　　D) 1.33 goals　　　　E) NOTA

6. Find the sum of x and y to the nearest hundredth using the following equations:
$$x - 5y = 9$$
$$8y + 5x = 12$$

A) 3.00　　　　B) 1.00　　　　C) 2.29　　　　D) 1.24　　　　E) NOTA

7. Let the operation $A \otimes B$ be defined as $A \otimes B = 2A^2 - 3B$. Find $(3 \otimes 4) \otimes 5$.

A) 60　　　　B) 57　　　　C) 15　　　　D) 12　　　　E) NOTA

8. George and Mary Smith are expecting twins. They already have 3 girls and 1 boy. What is the chance that the first twin born with be a boy?

A) ½   B) 2/5   C) 1/3   D) 2/3   E) NOTA

9. Solve for x: $|5x - 12| > 8$

A) x > 4   B) 4 > x > 4/5   C) x > 4 OR x < 4/5   D) x = 4/5   E) NOTA

10. Solve for x: $8x - 11 = 22x - 5^2$

A) x = 2   B) x = ¾   C) x = 14   D) x = 4   E) NOTA

11. Rikin has 2 snakes. One is 26 inches long and grows at the rate of 2 inches per month. The other is 18 inches and grows at a rate of 1 inch per week. How many weeks until they will be the same size?

A) 6   B) 8   C) 14   D) 16   E) NOTA

12. The number of kiwi is three less than twice the number of mangos. If there is a total of 51 items, how many more kiwi are there than mangos?

A) 15   B) 33   C) 18   D) 3   E) NOTA

13. Solve for x: $4^{x-3} = 64$

A) x = 3   B) x = 6   C) x = 9   D) x = 16   E) NOTA

14. Which of the following are rational numbers?
   I. $\dfrac{\sqrt{4}}{\sqrt{144}}$
   II. $\sqrt{2}$
   III.   -1/2
   IV.   0

A) III only   B) I, II, and III   C. I, III, and IV   D: I, II, III, and IV   E. NOTA

15. Given: $\dfrac{a}{b} = \dfrac{c}{d}$

Let a = 621, b = 63, and c = 3105. Find d.

A) 243   B) 49.3   C) 5   D) 315   E) NOTA

16. What is the sum of all real values of x to the nearest tenth in the given equation?
$4x^2 - 5x + 6 = 7$

A) 1   B) 1.2   C) 1.3   D) 3.4   E) NOTA

17. The units digit of a two-digit number is 3 times the tens digit, and the number is 30 more than the units digit. Find the product of the digits.

A) 13   B) 27   C) 39   D) 30   E) NOTA

18. If $(x + y)^2 = 81$ and xy = 18, then $x^2 + y^2 = ?$

A) 45   B) 53   C) 65   D) 81   E) NOTA

19. A plane takes one hour to go 350 miles with the wind. Flying into the wind, the plane covers only 300 miles in one hour. What is the speed of the wind?

A) 50 mph   B) 35 mph   C) 25 mph   D) 15 mph   E) NOTA

20. Jill is 4 years younger than Laurie. If Jill's age is doubled, it is 8 years greater than Laurie's age. What is the sum of Jill and Laurie's ages?

A) 12   B) 14   C) 24   D) 28   E) NOTA

21. How many of the following equations have the same y-intercept?

        I. y = 2x – 4   II. x = 5y + 10
        III. x + y = 8   IV. 3x – y = 2

A) 0   B) 2   C) 3   D) 4   E) NOTA

22. Which of the following expressions is equivalent to: $\dfrac{10 + 5\sqrt{24} + \sqrt{125} - 2\sqrt{25}}{\sqrt{200}}$?

A) $\dfrac{2\sqrt{3} + 5\sqrt{5}}{10\sqrt{2}}$ B) $\dfrac{\sqrt{6} + \sqrt{5}}{2\sqrt{2}}$ C) $\dfrac{2\sqrt{6} + \sqrt{5}}{2\sqrt{2}}$ D) $\dfrac{\sqrt{3} + \sqrt{5}}{\sqrt{2}}$ E) NOTA

23. In Zach's Zoo of Zooming Animals, there are two different species: ostriches and lions. Zach has Zookeeper Zan go around and count how many of each animal there are in total. Zan spots 36 eyes and 56 legs. How many more lions are there than ostriches?

A) 5   B) 10   C) 3   D) 2   E) NOTA

24. The train from Tampa to Orlando leaves at 1:00 PM. It must travel 90 miles. When the train that left Tampa is halfway to its destination, a Ferrari gets onto I-4 between Orlando and Tampa. If the Ferrari has to travel 50 miles of the freeway to get to Orlando, and the train is traveling at 150 miles per hour, how fast is the Ferrari's average speed (to the nearest tenth, in mph) if both arrive at the train station at the same time?

A) 166.6 mph B) 166.7 mph C) 83.3 mph   D) 83.4 mph   E) NOTA

25. It takes Kara 15 minutes to get to school from her house. She travels at 45 mph the entire trip. It takes Akash 25 minutes to get to school from his house, but he travels at a constant rate of 40 mph. What is the difference of the lengths from their houses to school to the nearest hundredth of a mile?

A) 5.42 miles   B) 10 miles   C) 5 miles   D) 10. 32 miles   E) NOTA

26. There is a total of $2.69 in a bag of coins. Every coin is either a quarter, dime, nickel, or penny. Out of the 30 total coins, 1/5 are quarters. The number of dimes is equal to the number of nickels plus the number of pennies, minus 10. $1.10 of the total is of either nickels or dimes. How many nickels are there?

A) 6   B) 7   C) 8   D) 9   E) NOTA

27. Find the y intercept of the line that is perpendicular to the line y = 2x – 7 and passes through the point (3, 5).

A) -7   B) 1/7   C) 3.5   D) -3.5   E) NOTA

28. Vibha opened a bank account with 4.5% return. After one year, she had not added or withdrawn any money from the account. With the interest she made, her new balance was $1881. How much money did Vibha put in her account at the beginning of the year?

A) $1796.36   B) $1800.00   C) $1719.00   D) $1852.37   E) NOTA

29. It takes Raj 5 hours to paint a house, and it takes Channing 4 to paint the same one. How long will it take them if they work together (rounded to the nearest hundredth of an hour)?

A) 4.50 hours B) 2.25 hours C) 2.22 hours D) 1.11 hours E) NOTA

30. If $f(x) = 3x^2 + x - 4$, what does f(4) equal?

A) 1   B) 4/3   C) 24   D) 48   E) NOTA

The ages of three trees total exactly 1000 years. When the youngest tree reaches the age of the middle tree, the middle tree will be the age of the oldest tree and four times the current age of the youngest. What is the exact age of the oldest tree?

Solve for x in the following equations:

A: $2x^2 + 12x + 18 = 0$　　　　　　　　　　B: $4x^2 - 100 = 0$
C: $6x^2 + 5x = 4$　　　　　　　　　　　　　D: $x^2 + 14x + 45 = 0$

Find the product of the GREATEST values of x from each equation.

A: Find the slope of the line that passes through (14, 16) and (5, 9).

B: Find the x intercept of the line $y = 7x - 6$

C: Find the y-intercept of the line that is parallel to $y = 3x + 11$ and passes through the point (2, 1)

D: Find the slope of the equation $4y + 6x = 14$

FIND $\dfrac{A - B}{C + D}$ as a fraction reduced to lowest terms.

Kyle goes canoeing with his friends every Saturday at exactly 10 am. Today, he is paddling UPSTREAM at a rate of 6 m/s. He is trying to meet up with Taylor who is anchored 448 meters away. How long will it take Kyle to reach Taylor if the current of the water is 2 m/s? Express answer in minutes and seconds.

Pete the peacock and Erin the emu are both eating kernels of corn. If x represents the number of minutes that Pete and Erin have been pecking and if y represents the number of dozens of kernels they have eaten, then Pete's function is given by the equation $y = 4x^2 -2$, and Erin's function is given by the equation $y = 3x^2 +2x – 7$. After 4 minutes, how many more kernels has Pete eaten than Erin?

Every Wednesday is Chik-fil-a Day for a group of friends. Scott's total cost is always $4.39. Stacey and Emily split large fries and each get a drink, costing them $3.98 together. Stephen and Leah each get chicken sandwiches and drinks, and pay a total of $2.76 each. Kara's sweet tea and ice cream cost $1.80. (NOTE: tax is already included. Assume that a 10% reduction is the bill will result in a 10% reduction in the tax paid.)

A: If Chik-fil-a Joey gives them a coupon for 10% off their total, what will the group's total come to all together to the nearest cent?

B: If they then split the cost equally among themselves, how much will each person have to pay (to the nearest cent)?

Find to the nearest penny, the arithmetic mean (average) of A and B.

Every student in IB must complete 200 hours of volunteering. There are 140 seniors in the program. The number of hours logged by each student per week varies from 2 to 15. By the beginning of the senior year, 40% of the students are done with their hours. Another 30% are exactly half way done, 15% are one quarter of the way done, and the final 15% have not started. How many hours does the entire class have left to complete?

WHAT NUMBER AM I? Find the numbers that are described by each clue below.

A: I am the only two-digit number that is both a square and a cube.

B: I am the only three-digit number that is both a square and a cube.

C: I am a perfect square, and the sum of the two perfect squares that come just before me.

FIND (B – A) to the nearest tenth.
          C

A: Find the GCF of 152 and 176
B: Find the LCM of 14 and 22
C: Find the greatest prime factor of 76
D: find the arithmetic mean of the median and range of following set of numbers:
        {16, 43, 29, 58, 23, 28, 9, 17}

Find $(A + B) \times (D - C)$

After every Mu Alpha Theta competition, Greg, Cody, and Mike go out for pizza. Each orders either mushroom or sausage. If Greg orders mushroom, then Cody orders what Mike orders. If Cody orders mushroom, then Greg orders the pizza that Mike doesn't order. If Mike orders sausage, then Greg orders the pizza that Cody orders. Who always gets the same kind of pizza?

If $f(x) = 2x^2 - 1$, $g(x) = x + 4$, and $h(x) = x/2$, what does $f(g(h(4)))$ equal?

On Keegan's last birthday, he weighed 145 lbs. Rob weighs 1/5 less than Keegan does and James weighs 3/4 more than Rob. What is the weight difference between Keegan and James? Give your answer to the nearest tenths digit.

Write x in terms of y:

$$y = \frac{2(4x + 17)}{9}$$

Algebra I Individual Answers:

1. c
2. c
3. d
4. a
5. d
6. a
7. b
8. a
9. c
10. e
11. d
12. a
13. b
14. c
15. d
16. b
17. b
18. a
19. c
20. d
21. b
22. c
23. d
24. b
25. a
26. c
27. e
28. b
29. c
30. d

Algebra I Team Answers

1. 533 years 4 months or $533\frac{1}{3}$ years
2. 37.5
3. 10/819
4. 1 min 52 sec
5. 156 kernels or 13 dozen kernels
6. $8.24
7. 11550
8. 26.6
9. 2956.5
10. Greg
11. 71
12. 58 lbs
13. 9y/8 -17/4 = x

Individual solutions:

1. **C** $z = 84$ so if $6y = 7z$ then:
$$6y = 7(84)$$
$$6y = 588$$
$$y = 98 \qquad y = 98 \text{ so if } 8x = 4y \text{ then:}$$
$$8x = 4(98)$$
$$8x = 392$$
$$x = 49 \qquad x = 49 \text{ so if } 7w = 2x \text{ then:}$$
$$7w = 2(49)$$
$$7w = 98$$
$$w = 14$$

$w + x + y + z = 14 + 49 + 98 + 84 = 245$

2. **C** $(3,7), (6,10)$

Slope $= \dfrac{Y_2 - Y_1}{X_2 - X_1} = \dfrac{10 - 7}{6 - 3} = \dfrac{3}{3} = 1$

Slope of line perpendicular $= -1/m$ where m is the slope

$\underline{-1/(1) = -1\quad}$

3. **D** Quadratic formula $\Rightarrow x = \dfrac{13 \pm \sqrt{169 - 4(2)(21)}}{2(2)}$

$x = 7/2, 3 \qquad$ Mean $= \dfrac{(7/2) + 3}{2} = 13/4$

4. **A** $\dfrac{(2x^{-3}y^4z^{-2})^{-1}}{3x^2y^{-1}z^5} = \dfrac{1}{3x^2y^{-1} \cdot 2x^{-3}y^4z^{-2}} = \dfrac{yx^3z^2}{6x^2z^3y^4} = \dfrac{x}{6y^3z^3}$

5. **D** $.4x = 8 \rightarrow$ x= number of shots taken

$x = 20$

increase to 50% $\rightarrow$ $\dfrac{8 \text{ goals}}{20 \text{ shots}}$ increase to $\dfrac{12 \text{ goals }}{24 \text{ shots}}$ (50%)

4 goals in 3 games needed, or 1.33 goals per game.

6. **A** $(x - 5y = 9)$ x -5 $\leftarrow$ solve using systems of equations.

$5x + 8y = 12$

$$-5x + 25y = -45$$
$$+\underline{5x + 8y = 12}$$
$$33y = -33$$
$$y = -1$$
$$x = 4 \qquad \text{Sum} = -1 + 4 = 3$$

7. **B** $A \otimes B = 2A^2 - 3B$

$3 \otimes 4 = 2(3)^2 - 3(4) = 6$

$6 \otimes 5 = 2(6)^2 - 3(5) = 57$

8. **A** each child is independent so there is a 50% chance that the first will be a boy $\rightarrow$ ½

9. **C** $|5x - 12| > 8$

$$5x - 12 > 8 \qquad\qquad 5x - 12 < -8$$
$$5x > 20 \qquad\qquad 5x < 4$$
$$x > 4 \qquad\qquad x < 4/5$$

10. **E** $8x - 11 = 22x - 5^2$

$$8x - 11 = 22x - 25$$
$$14 = 14x$$
$$x = 1$$

11. **D** S1 is 26 inches and grows ½ inch per week
   S2 is 18 inches and grows 1 inch per week
   $$26 + 1/2w = 18 + 1w$$
   $$8 = 1/2w$$
   $$16 = w$$

12. **A** $K = 2M - 3$
   $K + M = 51$ → substitution
   $2M - 3 + M = 51$
   $3M - 3 = 51$
   $3M = 54$
   $M = 18$
$K + M = 51$
$K + 18 = 51$
$K = 33$
   $K - M = 33-18 = 15$

13. **B** $4^{x-3} = 64$
   $4^3 = 64$
$x - 3 = 3$
$x = 6$

14. **C** I. 2/12 Rational II. $\sqrt{2}$ Irrational III. -1/2 Rational IV. 0 Rational

15. **D** $\dfrac{621}{63} = \dfrac{3105}{D}$

$d = 315$

16. **B** Quadratic formula → $4x^2 - 5x -1 = 0$
$$\frac{5 \pm \sqrt{25 - 4(4)(-1)}}{8} = x$$
                              $1.425, -.175 = x$ sum: 1.2

17. **B** 39; 3 X 9 = 27

18. **A** $(x + y)^2 = 81$
   $x + y = 9$
   $x = 9 - y$ $xy = 18$
                  $(9-y)(y) = 18$
                  $9y - y^2 = 18$
                     $y = 3$
                     $x = 6$
                  $6^2 + 3^2 = 45$

19. **C** p = plane speed
   w = wind speed
p + w = 350
p – w = 300 → combine   → 2w = 50
                           w = 25

20. **D** $J = L - 4$
   $2J = L + 8$

Combine:
     -J = -12
     J = 12
     L = 16
     L + J = 28

21. **B** y-intercept is when x = 0

    I. $y = 2(0) - 4$ II. $0 = 5y + 10$
        $Y = -4$ $y = -2$
    III. $0 + y = 8$ IV. $3(0) - y = 2$
        $Y = 8$ $y = -2$ II and IV

22. **C** $\dfrac{10 + 5\sqrt{24} + \sqrt{125} - 2\sqrt{25}}{\sqrt{200}} = \dfrac{10 + 5\sqrt{6\cdot4} + \sqrt{5\cdot25} - 2\sqrt{25}}{\sqrt{2\cdot100}}$

$= \dfrac{10 + 5\cdot2\sqrt{6} + 5\sqrt{5} - 2\cdot5}{10\sqrt{2}}$

simplify and divide by 5: $\dfrac{2\sqrt{6} + \sqrt{5}}{2\sqrt{2}}$

23. **D** $2L + 2O = 36$ ← each animal has two eyes
    $4L + 2O = 56$ ← lions have four legs and ostriches have two.
Combination gives us
     $2L = 20$
     $L = 10$
     $O = 8$
        $L - O = 10-8 = 2$

24. **B** Train: 45 miles to go at 150 mph → .3 hours to reach Orlando
    Car: 50 miles to go in .3 hours → 50/.3 hrs = speed = 166.7

25. **A** Kara: 15 minutes = .25 hours → .25 x 45 mph = 11.25 miles
    Akash: 25 minutes = 5/12 hours → 5/12 x 40 mph = 16. 67 miles
        16.67 -11.25 = 5.42 miles

26. **C** $1/5(30) = Q$
     $6 = Q$ $D = N + P - 10$
                    $1.1 = .05N + .1D$
                    $D + Q + N + P = 30$

$\$2.69 - 6(.25) = \$1.19$ ← total after quarters
         - $\underline{\$1.10}$ ← amount of nickels or dimes
          $\$ .09$ ← nine pennies

         $D + Q + N + P = 30$
         $D + 6 + N + 9 = 30$
         $D + N = 15$
         $.1D + .05N = 1.1$ ← combination $.5 N = 4$
                                     $N = 8$

27. **E** perpendicular→ slope = -1/(2) = -1/2
                      $y = -x/2 + b$
                      $5 = -(3)/2 + b$
                      $B = 6.5$

28. **B** x = amount put in

1.045x = 1881
    x = 1800

29. **C** R→ 5 hours per house OR 1/5 house per hour
C→ 4 hours per house OR ¼ house per hour
    1/5 + ¼ = amount of house the two can paint in one hour
    9/20 = amount of house per hour
    20/9= hours to paint house
    = 2.22

30. **D** $f(x) = 3x^2 + x - 4$
    $f(4) = 3(4)^2 + 4 - 4$
    $f(4) = 48$

Team Round Solutions

1. a + b + c = 1000
    b – a = c – b
    c = 4a

    Substitute: a + b + 4a = 1000
    $\qquad\qquad$ 5a + b = 1000

    $\qquad\qquad$ b – a = 4a – b
    $\qquad\qquad$ b = 5a – b
    $\qquad\qquad$ b = 5/2 a

    $\qquad$ a + 5a/2 + 4a = 1000
    $\qquad\qquad$ a = 133 1/3
    $\qquad\qquad$ c = 4a
    $\qquad\qquad$ c = 4(133 1/3)
    $\qquad\qquad$ **c = 533.3** years OR 533 years and 4 months

2. A. x = $\dfrac{-12 \pm \sqrt{144 - 4(2)(18)}}{4}$ = -3

    B. x = $\dfrac{\pm \sqrt{-4(4)(-100)}}{8}$ = 5, -5

    C. x = $\dfrac{-5 \pm \sqrt{25 - 4(6)(-4)}}{12}$ = ½, -4/3

    D. x = $\dfrac{-14 \pm \sqrt{196 - 4(1)(45)}}{2}$ = -5, -9

    -3 x 5 x ½ x -5 = **37.5**

3. A. $\dfrac{Y2 - Y1}{X2 - X1} = \dfrac{9 - 16}{5 - 14} = \dfrac{7}{9}$

   B. y = 7x – 6
      0 = 7x -6
      6/7 = x

   C. Slope = 3
      Y = 3x + 6
      1 = 3(2) + b
      1 = 6 + b
      -5 = b

   D. 4y + 6x = 14
      Put in slope intercept form: y = -3x/2 + 7/2
      Slope = -3/2

   $\dfrac{A - B}{C + D} = \dfrac{7/9 - 6/7}{-5 + -3/2} = \dfrac{\mathbf{10}}{\mathbf{819}}$

4. 6 m/s – 2 m/s = 4 m/s $\qquad$ (448 m)/ (4 m/s) = 112 sec = **1 minute 52 seconds**

5. $(4(4)^2 - 2) - (3(4)^2 + 2(4) - 7) = 13$

         Measured in dozens: 13 x 12 = **156 kernels or 13 dozen**

6. A. .9(4.39 + 3.98 + 2[2.76] + 1.80) = $14.12

     B. $14.12/6 = $2.35

        Mean: $\dfrac{14.12 + 2.35}{2}$ = $8.235 or rounded, **$8.24**

7. [(.3 x 100 hours left) + (.15 x 150 hours left) + (.15 x 200 hours left)] x 140 students
                                     = 11550 hours

8. A. 64

     B. 729

     C. 25

               $\dfrac{729 - 64}{25}$ = **26.6**

9. A. 8

     B. 154

     C. 19

     D. median = 25.5

       Range = 49

       Mean of median and range = 37.25

     (8 + 154) X (37.25 − 19) = **2956.5**

10. Apply each condition to each person and it can be determined that only Greg will always order the same kind of pizza. **Greg**

11. h(4) = 4/2 = 2

     g(2) = 2 + 4 = 6

     f(6) = $2(6)^2$ − 1 = **71**

12. K = 145

     R = 145 − 1/5(145) = 116

     J = 116 + ¾(116) = 203

       J − K = 203 − 145 = **58 lbs**

13. y = $\dfrac{2(4x + 17)}{9}$

       9y = 8x + 34

       9y − 34 = 8x

       9y/8 − 17/4 = x