January Regional Calculus Exam

Individual

NOTA is defined as None of the Above Answers is Correct

1) Find a real number integer value of x that gives a vertical tangent line to the function $f(x) = e^{\sqrt[3]{x-1}}$.

A) -1 B) 0 C) 1 D) e E) NOTA

2) Using four rectangles on a regular partition of [0,2] calculate the **lower** sum approximation of $\int_0^2 (3x^2 + 2)dx$.

A) $\frac{37}{2}$ B) $\frac{37}{4}$ C) $\frac{61}{2}$ D) $\frac{61}{4}$ E) NOTA

3) Find the equation of the tangent line to $y = \arctan(2x)$ at $x = 0$.

A) $y = -2x$ B) $y = -4x$ C) $y = -4x + 2$ D) $y = 4x$ E) NOTA

4) If $f_0(x) = \left(\frac{x}{x+1}\right)$ and $f_{(n+1)} = f_0 \circ f_n$ for $n = (0,1,2...)$ Solve for $f_{49}(12)$. Put your answer in decimal form, **do not round,** and give the integer value of the 5th decimal place.

A) 3 B) 6 C) 7 D) 8 E) NOTA

5) F(x) is defined to be $x^2y'' - 4xy' + 6y = 0$. Which of the following equations are solutions to F(x)?

I. $y_1 = x^2$

II. $y_2 = x^2 + 2$

III. $y_3 = x^3$

IV. $y_4 = x^4$

A) I & II only
B) I & IV only
C) III only
D) I, III, & IV only
E) NOTA

6) The linear distance between Point A and Point B is 315 meters. At time t_1, A and B move directly towards each other until they meet at t_2. A third Point C starts in the same position as A and travels directly to B, then continues to go back and forth between A and B until A and B meet. Find the total distance traveled by C (in meters) on the interval of time $[t_1, t_2]$, given:
Magnitude of A's velocity: 35 m/s
Magnitude of B's velocity: 10 m/s
Magnitude of C's velocity: 1254 m/s

A) 315 B) 1254 C) 2205 D) 8778 E) NOTA

7) Given the derivatives $h' = 3\cos(x)\sqrt{\sin(x)}$ and $g' = \frac{1-x}{g^2}$ integrate to find the original equations of each using the initial conditions $h(\pi) = 4$, and $g(0) = 2$. Solve for h and g at $x = \frac{\sqrt{3}}{2}$. Your final answer is given by $W(x) = hg$, round to the nearest whole number.

A) 0 B) 11 C) 17 D) 35 E) NOTA

8) Given $y = C_1e^{-x} + C_2e^{x} + C_3e^{2x}$ where C_1, C_2, C_3 are arbitrary constants. Using the following system of equations, find y (to the nearest whole number) given that $x = 1$:

$$C_1'e^{-x} + C_2'e^{x} + C_3'e^{2x} = 0$$
$$-C_1'e^{-x} + C_2'e^{x} + 2C_3'e^{2x} = 0$$
$$C_1'e^{-x} + C_2'e^{x} + 4C_3'e^{2x} = e^{5x}$$

A) 0 B) 1 C) 2 D) −72 E) NOTA

9) Let $W(y_1, y_2, y_3 \ldots y_n) = Det\begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ y_1'' & y_2'' & \cdots & y_n'' \\ \vdots & \vdots & & \vdots \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}$.

Given $y_1 = x; y_2 = x^2; y_3 = x^{-1}$. Find W to the nearest whole number when $x = 2$.

A) -3 B) -1 C) 3 D) 6 E) NOTA

10) Find the derivative of $y=[\tan^{-1}(\tan^{-1}(\tan^{-1}(x^2))]$ at $x=\dfrac{\pi}{\sqrt{2}}$. Give the thousandth digits as your answer.

A) 2 B) 3 C) 4 D) 5 E) NOTA

Questions 11 & 12 refer to Figure 1, below.

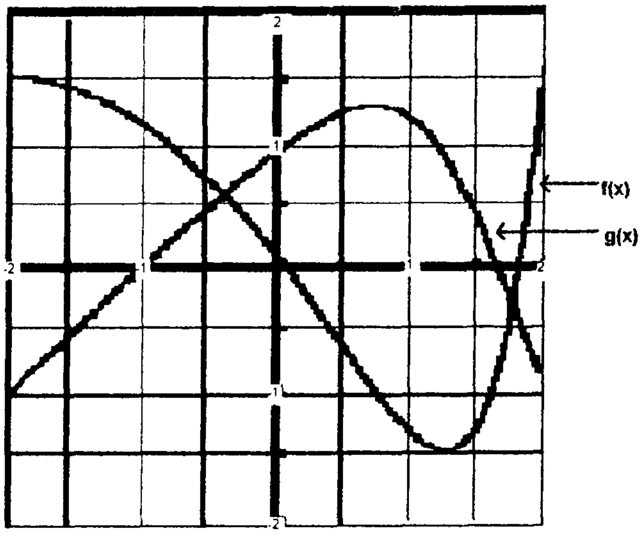


Figure 1

11) Define a function $h=\dfrac{\sqrt[3]{[f\circ g(x)]^2}}{-g'(x-1.7)}$ and estimate the value of $h\left(\dfrac{7}{10}\right)$. Round to the hundredth digit.

A) -.79 B) -.97 C) -1.31 D) 2.63 E)NOTA

12) True or False. For all true statements sum the integer values with in the parenthesis (next to the respective statement) to get the final answer. Please note: if the statement is false it is not included in your integer sum.

I. (3) The second derivative of $f(x)$ is negative on the interval of x values $\left(\dfrac{1}{2},\dfrac{13}{10}\right)$.

II. (7) $g\circ f(x)$ has a local maximum on the interval on the interval of x values $\left(-1,\dfrac{-2}{10}\right)$.

III. (5) The slopes of $g(x)$ and $f(x)$ are equal at some value x in the interval of x values $\left(-\dfrac{1}{2},\dfrac{1}{2}\right)$.

A) 3 B) 7 C) 10 D) 15 E) NOTA

13) Use differentials to approximate $\tan\left(\frac{29\pi}{192}\right)$, given that $\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$. Give the fourth integer right of the decimal place.

A) 2 B) 5 C) 6 D) 9 E) NOTA

14) Solve: $\int_0^2\left(\frac{4x^3+2x+1}{2x^4+2x^2+2x+1}\right)dx$. Give an exact answer.

A) $\frac{\ln(2)}{2}$ B) $\ln(2)$ C) $\frac{\ln(45)}{4}$ D) $4\ln(45)$ E) NOTA

15) Find the 112^{th} derivative of ln(x) at x=1 and divide this by 109!. Use only two significant figures in your answer.

A) -12000 B) $-13\text{x}10^6$ C) 73000 D) 12000 E) NOTA

16) Mikey has worked at a summer camp for 3 consecutive years. The first year he worked there were 100 total campers by the third year there were 300 total campers in the program. Assuming the number of campers is increasing exponentially, how many more years will Mikey have to work before there are 1000 campers?

A) 3 B) 4 C) 5 D) 7 E) NOTA

17) If $f(18)=5$ and $f'(18)=17$ use a linear approximation at $x=18$ to determine an approximation for $f(17.9)$. Give answer to the tenth place.

A) 2.0 B) 3.3 C) 5 D) 16.5 E) NOTA

18) The sum of two non negative real numbers x and y is equal to 108. What is the units digit of largest possible product of x^2 and y?

A) 2 B) 4 C) 7 D) 9 E) NOTA

January Regional Calculus Exam

19) Given $f(x)=\begin{cases} \dfrac{x^3+5x^2-38x-168}{x-6}, & x \neq 6 \\ k & x=6 \end{cases}$ What value of k makes f continuous on the interval $(-\infty,\infty)$?

A) -25 B) 6 C) 120 D) 130 E) NOTA

20) The radius of a circle is increasing at a rate of k, $k>0$.
Let r_1 = the radius of the circle where the rate of increase of the area of the circle is 2 times the rate of increase of the circumference.
Let r_2 = the radius of the circle where that rate of increase of the area of the circle is 5 times the rate of increase of the circumference.
Find $(r_1+r_2)r_1$.

A) 3.5 B) 7 C) 14 D) 160 E) NOTA

21) Consider $f(x)=\frac{1}{6}x^6+\frac{8}{5}x^5+3x^4-\frac{22}{3}x^3-\frac{29}{2}x^2+30x$ for values of x from (-3,3) find the sum of the x-coordinates of all the critical points on this interval.

A) -8 B) -3 C) -1 D) 0 E) NOTA

22) Solve $\lim_{x\to 0}[1-\sin(3x)]^{1/x}$. Give the hundredth digit of your answer. (Round your answer to the hundredth place).

A) 0 B) 1 C) 3 D) 5 E) NOTA

23) If $y=x^{x^{x^{x}}}$, find $\frac{dx}{dy}$ evaluated where $x=\sqrt{2}$. Round to the hundredths digit.

A) .08 B) .11 C) .22 D) .43 E) NOTA

24) Find the product of the magnitude of all possible tangent segments drawn from the point (1,-5) to the graph of $y=x^3+2x-7$. Round to the nearest whole number.

A) 5 B) 17 C) 20 D) 44 E) NOTA

January Regional Calculus Exam

25) Given $h(x) = f(g(x))$ and $g(1) = 3$; $g'(3) = 5$; $f(1) = 3$; $f'(3) = 13$. Find $\dfrac{h'(x)}{g'(x)}$ at $x = 1$.

A) 3 B) 5 C) 9 D) 13 E) NOTA

26) Find $\lim_{h \to 0} \dfrac{e^{2h+2} - e^2}{h}$ at the point $x = 1$. Round to the hundredth digit.

A) 2.72 B) 5.44 C) 7.39 D) 14.78 E) NOTA

27) Find the area under the curve $f(x) = \sin(x) + 2$ on the interval of x values $[0, \frac{\pi}{3}]$. Give answer to two decimal places.

A) 1.55 B) 2.09 C) 2.59 D) 6.78 E) NOTA

28) An Isosceles triangle is inscribed inside a parabola $y = x^2$ with one vertex of the triangle at the vertex of the parabola and the other two vertices lie on the parabola perpendicular to the parabola's axis. How fast is the area of the triangle changing when its height is 2 units if the height is increasing at a rate of 12 units per second?

A) $9\sqrt{2}$ B) 18 C) $18\sqrt{2}$ D) 36 E) NOTA

29) Solve for $y(2.2)$ given that $y = \int \frac{x}{e^x} dx$ and $y(2) = 4$. Give your answer to two decimal places.

A) .81 B) 4.05 C) 4.41 D) 4.81 E) NOTA

30) $\dfrac{dy}{dx} = \dfrac{1 - x^2}{y^2}$ Given $y(3)=0$ Find $y(1)$. Give two decimal places.

A) 1.39 B) 2.71 C) 2.88 D) 20 E) NOTA

1) Mr. Frazer wishes to make a topless box from a flat piece of cardboard with dimensions 45 ft by 33 ft by cutting squares from the corners and then folding the cardboard to form a box. Give the area of one of the removed squares in square feet, which maximizes the volume of his box. (Give your answer to the nearest square foot)

2) To the nearest unit, find

$$\left(\lim_{x\to 1}\frac{\sin(x-1)}{x-1}\right)+\left(\lim_{x\to 3}2\right)+\left(\lim_{x\to\infty}\frac{4x^2+5x-2}{x(2x^2+5x-2)}\right)+\left(\lim_{x\to 0}\frac{\cos x-1}{x}\right)$$

3) Daniel is 100 ft due North of Russell at $t=0$ seconds. Daniel travels north at 10 ft/s, and Russell travels southwest at 15 ft/s. How fast is the distance between Daniel and Russell changing $t=5$? (Give two decimal places in answer).

4) Evaluate $\int_{-5}^{3}\left|x^3-x\right|dx$ (Give an exact answer).

5) How fast is the surface area of a sphere increasing when the radius is $\frac{7}{5}$ meters given that the volume is increasing at a rate of $10\frac{m^3}{s}$? (Give an exact answer in meters^2 per second)

6) The average value of $f(x)=3x^2+16x-29$ is 157 on the interval $[5, v]$ where $v>5$. Find v. (Give an exact answer).

7) Let $f(x)=\cos(x)$ and $g(x)=\sin(x)$

Let A= the area bounded by the curves between $x=0$ and $x=\frac{\pi}{4}$.

Let B= the volume of the solid obtained by rotating the region in A about the x axis.

Let C= $\sqrt{g\circ f\circ g\left(\frac{-\pi}{4}\right)}$

Let D= The value of x where the two functions intersect for the fourth time to the right of the y-axis.

Find $\frac{A+B}{C(C+D)}$. Give an approximate answer (rounded to the nearest hundredth digit).

8) Find the slope of the normal line to $y=x+\cos(xy)$ when $x=0$.

9) Using 6 rectangles on a regular partition of [0,15] calculate the lower sum approximation of $\int_{0}^{15}\frac{\ln(x+1)}{2^x}$ dx. Sum the first 3 digits to the right of the decimal place.

10) The line $16x+y-5=0$ is normal to the curve $y=x^4+k$. Give the exact value of k.

11. One cone is inscribed within another one. The outer cone has height 10 and radius of 4. The inner cone is inscribed so that its apex lies on the base of the outer cone. The base of the inner cone is parallel to the base of the outer cone. The axes of the cones are collinear. Find the maximum volume of the inner cone. (Give an exact answer).

12) A function is given by $f(x)=x^5-5x^4-143x^3+125x^2+4406x+7280$.
Let A= the sum of all the zeros of $f(x)$.
Let B= the area under the parabola $g(x)=x^2$ bounded by the the x-axis, and the line $x=d$, where (d,h) is the point of intersection of $f(x)$ and g(x). Give your answer to the nearest integer.
Let C= The value of x which gives the largest local maximum value for $f(x)$. Save 2 decimal places.
Your answer is given as: $A+B+C$

13) A line and a parabola pass through the origin. The line is tangent to the parabola at the origin. The parabola reaches a maximum height of k and also passes through the point (2h,0). Find the intersection of the line and the axis of symmetry of the parabola in terms of h and k.

14) Given two ellipses the smaller one within the larger one with exactly overlapping minor and major axes (though the major axis and minor axis of the larger ellipse will extend past that of the minor one). Given that the larger ellipse has a minor axis of 6 ft and the major axis 12ft, and the major axis of the larger ellipse extends past the major axis of the minor ellipse by 1 ft on either side. Calculate the area of the major ellipse minus that of the smaller one. Give an exact answer in terms of feet^2.

15) Andy buys a cookie with a diameter of 12 inches. He wishes to cut the cookie so that his friend Nick will get twice as much of the cookie. He only makes one cut across the cookie... how long (in inches) is his single straight cut across the cookie (assuming that the cookie is a perfect circle with even thickness)? Round answer to hundredth place.

Mu Alpha Theta: Individual Solutions

Calculus

1) $f'(x) = \dfrac{e^{\sqrt[3]{x-1}}}{3(x-1)^{2/3}}$ and there is a vertical asymptote @ x=1

2) Length of each partition = .5 @ 0, .5, 1 & 1.5 ; heights are given by $3x^2+2 = 2, \frac{11}{4}, 5 \& \frac{35}{4}$ So the area is $\frac{1}{2}\left(2+\frac{11}{4}+5+\frac{35}{4}\right)=\frac{37}{4}$.

3) $y' = \dfrac{2}{4x^2+1}$ and $y'(0)=2$ @ (0,0) So the equation of the tangent line is $y=2x$.

4) Let us first consider the special cases n = 1, 2 and 3

$$f_1(x)=(f_0\circ f_0)(x)=f_0(f_0(x))=f_0\left(\frac{x}{x+1}\right)=\frac{\frac{x}{x+1}}{\frac{x}{x+1}+1}=\frac{x}{2x+1};f_2(x)=(f_0\circ f_1)(x)=f_0(f_1(x))=f_0\left(\frac{x}{2x+1}\right)=\frac{\frac{x}{2x+1}}{\frac{x}{2x+1}+1}=\frac{x}{3x+1}$$

$$f_3(x)=(f_0\circ f_2)(x)=f_0(f_2(x))=f_0\left(\frac{x}{3x+1}\right)=\frac{\frac{x}{3x+1}}{\frac{x}{3x+1}+1}=\frac{x}{4x+1}$$

So $f_n(x)=\dfrac{x}{(n+1)x+1}$ and $f_{49}(12)=\dfrac{12}{(49+1)12+1}$

5) We will consider each of the possible solutions separately.

I. $y_1=x^2$; $y_1'=2x$; $y_1''=2$. Consider $x^2(2)-4x(2x)+6(x^2)\equiv 0$, thus (I) is a solution.

II. $y_2=x^2+2$; $y_2'=2x$; $y_2''=2$. Consider $x^2(2)-4x(2x)+6(x^2+2)\neq 0$, thus (II) is **not** a solution.

III. $y_3=x^3$; $y_3'=3x^2$; $y_3''=6x$. Consider $x^2(6x)-4x(3x^2)+6(x^3)\equiv 0$, thus (III) is a solution.

IV. $y_4=x^4$; $y_4'=4x^3$; $y_4''=12x^2$. Consider $x^2(12x^2)-4x(4x^3)+6(x^4)\neq 0$, thus (IV) is **not** a solution.

6) First we find the total time it takes for A and B to meet. $10t+35t=315 \Rightarrow t=7$ so the total distance traveled by C is given by $(1254)(7)=8778$.

7) $\int h'(x)dx=\int 3\cos(x)\sqrt{\sin(x)}dx=2[\sin(x)]^{3/2}+C$

Impose the initial conditions $x=\pi, h=4$ so $h=2[\sin(x)]^{3/2}+4$. At $x=\frac{\sqrt{3}}{2}$, $h=5.3297$

$\int g':(1-x)dx=g^2dg,\ \frac{1}{3}g^3=x-\frac{x^2}{2}+C$

Impose the initial conditions $g(0)=2$, so $\frac{1}{3}g^3=x-\frac{x^2}{2}+\frac{8}{3}$ At $x=\frac{\sqrt{3}}{2}$, $g=2.115$

$W(x)=hg=11.272$

8) Solve for C_1', C_2', C_3'... Let $C_1'=\frac{D_1}{D}, C_2'=\frac{D_2}{D}, C_3'=\frac{D_3}{D}$

Then $D=(e^{-x})(e^{x})(e^{2x})Det\begin{vmatrix}1&1&1\\-1&1&2\\1&1&4\end{vmatrix}$, and $D_1=(e^{5x})(e^{x})(e^{2x})Det\begin{vmatrix}0&1&1\\0&1&2\\e^{5x}&1&4\end{vmatrix}$....

$C_1'=\frac{e^{6x}}{6}, C_2'=-\frac{e^{4x}}{2}$, and $C_3'=\frac{e^{3x}}{3}$.

Integrate C_1',C_2',C_3': $C_1=\frac{e^{6x}}{36}, C_2=-\frac{e^{4x}}{8}$, and $C_3=\frac{e^{3x}}{9}$. Replace C_1,C_2,C_3 into y and solve for x=1... Finally: $y=\frac{1}{72}e^{5x}=2.06$

9) $x\begin{vmatrix}2x&-x^{-2}\\2&2x^{-3}\end{vmatrix}-1\begin{vmatrix}x^2&x^{-1}\\2&2x^{-3}\end{vmatrix}+0\begin{vmatrix}x^2&x^{-1}\\2x&-x^{-2}\end{vmatrix}=6x^{-1}=3.$

10) $y'=\frac{2x}{(x^4+1)[(\tan^{-1}(x^2))^2+1][(\tan^{-1}(\tan^{-1}(x^2)))^2+1]}\approx .0323.$

11) $g(x)\approx 1.3$, $f(1.3)=-1.5$, $-g'(x-1.7)=-1$ Thus $\frac{\sqrt[3]{[f\circ g(x)]^2}}{-g'(x-1.7)}=-1.3103$

12) I. False ; II. True $g\circ f(x)$ has a local maximum at approximately ; -.5 III. False

13) Differential of $\tan(x)=\sec^2(x)dx$ Where $\sec^2(x)=\frac{4}{3}$ & $dx=\frac{-\pi}{64}$, so the approximation is given by $\frac{\sqrt{3}}{3}+\left(\frac{4}{3}\right)\left(\frac{-\pi}{64}\right)=0.5119004$

14) Let $u=2x^4+2x^2+2x+1$ and $\frac{1}{2}du=4x^3+2x+1$ The new limits of integration are 45 and 1.

So $\left(\frac{1}{2}\right)\int_1^{45}\frac{du}{u}=\frac{\ln(45)}{2}$.

15) Find the nth derivative of ln(x)

1 $1/x$

2 $-1/x^2$

3 $2/x^3$ So $\frac{(-1)^{113}(111!)}{(1)^{112}(109!)}=-12210$

4 $-6/x^4$

n $\frac{(-1)^{n+1}(n-1)!}{x^n}$

16) $1000=100e^{\left(\frac{\ln 3}{3}\right)t}$ t=6.2877 or 7 years minus 3= 4 more years.

17) $f(17.9)\approx f(18)-(0.1)(f'(18))=3.3$

18) $y=108-x$ and $M=x^2y$ so $M=x^2(108-x)=108x^2-x^3$. $M'=216x-3x^2=0$ So we test x=0, 72, and 108. So $(72)^2(36)$=186624

19) $f(x)=\dfrac{(x+7)(x+4)(x-6)}{(x-6)}=(x+7)(x+4)\Rightarrow f(6)=130=k$.

20) $\dfrac{dr}{dt}=k$, $A=\pi r^2\Rightarrow\dfrac{dA}{dt}=2\pi r\dfrac{dr}{dt}$ and $C=2\pi r\Rightarrow\dfrac{dC}{dt}=2\pi\dfrac{dr}{dt}$.

For r_1: $2\pi rk=4\pi k\Rightarrow r_1=2$; For r_2: $2\pi rk=10\pi k\Rightarrow r_2=5$

$(r_1+r_2)r_1=14$

21) (Recommend using synthetic division to find roots)

$f(x)=(x-1)^2(x+2)(x+3)(x+5)$. Values of c on the given interval: 1, 1, -2. So the sum of the possible value for c is -1.

22) $\lim_{x\to 0}[1-\sin(3x)]^{1/x}=y\Rightarrow\ln y=\lim_{x\to 0}\dfrac{\ln(1-\sin(3x))}{x}=\dfrac{-3\cos(3x)}{(1-\sin(3x))}=-3\Rightarrow y=e^{-3}$

23) $y=x^y\Rightarrow x=y^{1/y}$ and $\sqrt{2}=(2)^{1/2}=y^{1/y}\Rightarrow y=2$ so

$\ln(x)=\dfrac{\ln(y)}{y}\Rightarrow\dfrac{dx}{dy}=\left(\dfrac{1-\ln(y)}{y^2}\right)y^{1/y}\Rightarrow\dfrac{\sqrt{2}(1-\ln(2))}{4}$

24) $\dfrac{y+5}{x-1}=3x^2+2\Rightarrow y=3x^3-3x^2+2x-7=x^3+2x+7\Rightarrow 2x^3-3x^2=0\Rightarrow x=\dfrac{3}{2}\ \&\ 0$

(0,-7) and $\left(\dfrac{3}{2},\dfrac{-5}{8}\right)$. $D_1D_2=\left(\sqrt{5}\right)\left(\dfrac{\sqrt{1241}}{8}\right)=9.8465\approx 10$

25) $h'(x)=f'(g(x))g'(x)\Rightarrow\dfrac{h'(1)}{g'(1)}=f'(g(1))\Rightarrow f'(3)=13$.

26) $\lim_{h\to 0}\dfrac{e^{2h+2}-e^2}{h}=2e^{2x}\Rightarrow 2e^2$

27) $\displaystyle\int_0^{\pi/3}(\sin(x)+2)dx=\frac{2\pi}{3}+\frac{1}{2}$

28) $\dfrac{dh}{dt}=12$; $A=\dfrac{1}{2}bh=\dfrac{1}{2}bh=\dfrac{1}{2}*2*h\sqrt{h}=h^{3/2}$; $\dfrac{da}{dt}=\dfrac{3}{2}\sqrt{h}\dfrac{dh}{dt}=18\sqrt{2}$

29) $\displaystyle\int\left(\frac{x}{e^x}\right)dx=(-x-1)e^{-x}+C\Rightarrow y(2)=4;C=4.406\Rightarrow f(2.2)=4.05$

30) $\displaystyle\int(y^2)dy=\int(1-x^2)dx\Rightarrow\frac{y^3}{3}=x-\frac{x^3}{3}+C\Rightarrow C=6\Rightarrow y=\sqrt[3]{20}$

Calculus

Individual Questions

1. C
2. B
3. E
4. B
5. E
6. D
7. B
8. C
9. C
10. A
11. C
12. B
13. D
14. E
15. A
16. B
17. B
18. B
19. D
20. C
21. C
22. D
23. B
24. E
25. D
26. D
27. C
28. C
29. B
30. B

Team Solutions

1. 39
2. 3
3. 22.618
4. $\frac{321}{2}$
5. $\frac{100}{7}$
6. $\frac{\sqrt{653}-13}{2}$
7. .22
8. -1
9. 10
10. $\frac{255}{256}$
11. $\frac{640\pi}{81}$
12. 11.47
13. (h,2k)
14. 8π
15. 11.56

January Calculus Regional Team Solutions:

Question #1: $V = x(45-2x)(33-2x) = 4x^3 - 156x^2 + 1485x$ then

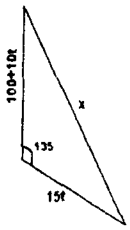
$V' = 12x^2 - 312x + 1485 = 0$ The roots are $\frac{26 \pm \sqrt{181}}{2}$. The smaller root produces the maximum volume, so the area of the square is $\left(\frac{26-\sqrt{181}}{2}\right)^2 = 39.353 \approx 39$.

Question #2: Let $n + a = b$ then $\lim_{x\to(b)^-} [x] = b-1$ and $\lim_{x\to(b)^-} |x| = b$, $\lim_{x\to(b)^+} [x] = b$ and $\lim_{x\to(b)^+} |x| = b \Rightarrow$

$$\left(\lim_{x\to(b)^-}(|x|-[x])\right) - \left(\lim_{x\to(b)^-}(|x|-[x])\right) + \left(\lim_{x\to(b)^+}(|x|-[x])\right) + \left(\lim_{x\to(b)}(|x|-[x])\right) =$$
$$(b-(b-1)) - (b-(b-1)) + (b-b) + (b-b) =$$
$$= 0$$

Question #3: $x^2 = (100+10t)^2 + (15t)^2 - 2(100+10t)(15t)(\cos 135) \Rightarrow$

$$2x\frac{dx}{dt} = 20(100+10t) + (30)(15t) + \sqrt{2}(1500+300t) \Rightarrow t = 5; x = 209.844; \frac{dx}{dt} = 22.618$$



Question #4: $\int_{-5}^{-1} -(x^3 - x)dx + \int_{-1}^{0}(x^3 - x)dx + \int_{0}^{1} -(x^3 - x)dx + \int_{1}^{3}(x^3 - x)dx = \frac{321}{2}$

Question #5: $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$; $A = 4\pi r^2 \Rightarrow \frac{dA}{dt} = 8\pi r^2 \frac{dr}{dt}$;

$$\frac{dV}{dt} = 10 = 4\pi\left(\frac{7}{5}\right)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{125}{98\pi}; \frac{dA}{dt} = 8\pi\left(\frac{125}{98\pi}\right)\left(\frac{7}{5}\right) = \frac{100}{7}$$

Question #6: $\frac{1}{v-2} = \int_5^v (3x^2 + 16x - 29)dx = \frac{1}{v-5}(v^3 + 8v^2 + 16v - 29) \Rightarrow v^2 + 13v + 36 = 157$.

$$v = \frac{\sqrt{653} - 13}{2}.$$

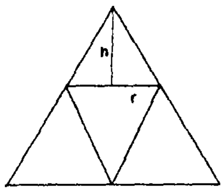
Question #7: $A=\int(\cos(x)-\sin(x))dx=\sqrt{2}-1$; $B=\pi\int_0^{\pi/4}(\cos^2(x)-\sin^2(x))dx=\frac{\pi}{2}$;

$C=\sqrt{\sin\left[\cos\left(\sin\left(\frac{-\pi}{4}\right)\right)\right]}$ =.8301197; $D=10.210176$ So the answer is given as .22

Question #8: This is a semicircle with radius 127 but we consider the radius to be 120 because that is the limit of integration so $\frac{1}{2}120^2\pi=22619.47$

Question #9: Heights occur at x = 0, 5, 7.5, 10, 12.5, 15
Area is 2.5(.07069)= .17672 sum is 14

Question #10: $4x^3=\frac{1}{16}\Rightarrow x=\frac{1}{4}$; $4+y-5=0\Rightarrow y=1$; $1=\frac{1}{256}+k\Rightarrow k=\frac{255}{256}$



Question #11: Solution:
Let r= radius of the inner cone
Let 10-h equal the height from the base of the inner cone to the vertex of the outer cone

Similar triangles shows that $\frac{10}{4}=\frac{h}{r}\Rightarrow\frac{2h}{5}=r$

Volume of the inner cone = $\frac{1}{3}\pi\left(\frac{2h}{5}\right)^2(10-h)$ Differentiate and set equal to 0 with gives h of the inner cone = $\frac{20}{3}$ So with $\frac{10}{4}=\frac{h}{r}$ then $r=\frac{8}{3}$. The maximum volume is

$$\frac{1}{3}\pi\left(\frac{8}{3}\right)^2\left(10-\frac{20}{3}\right)=\frac{1}{3}\pi\left(\frac{8}{3}\right)^2\left(\frac{10}{3}\right)=\frac{640\pi}{81}.$$

Question #12: A: $f(x)=x^5-5x^4-143x^3+125x^2+4406x+7280=0$ Sum of the zeros:
$13+7-2-5-8=5$

B: Intersection: -1.998357 $\int_{-1.998}^{0}x^2dx=2.661$

C: $\frac{d}{dx}\left(x^5-5x^4-143x^3+125x^2+4406x+7280\right)=5x^4-20x^3-429x^2+250x+4406$ Set equal to zero and find values of y at each critical point… at x=3.4689 $f(x)$ has a local maximum.

$A+B+C=11.13$

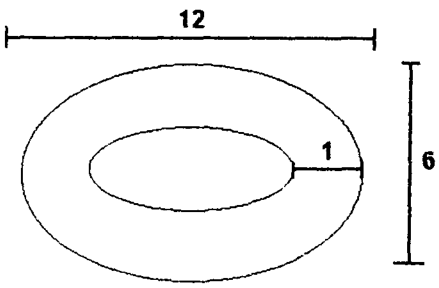
Question #13: The solutions of the parabola are 0 and 2h so the equation of the parabola is $(x-0)(x-2h)=0 \Rightarrow x^2-2hx=0$

The coordinate of the turning point of the parabola is $\left\{\frac{2h}{2},\left[\frac{4(-1)0-(2h)^2}{4}\right]\right\} \Rightarrow \left\{h,\frac{-h}{2}\right\}$

Axis of sym. $x=h$ and $k=-h^2$ So the equation becomes $y=\frac{-k}{h^2}(x-h)^2+k$ then using the formula about tangent lines we get $\frac{y}{2}=\frac{-k}{h^2}(x-h)(-h)+k$ then substitute $x=h$ giving $y=2k$ and then the point of intersection is $(h,2k)$

Question #14: The two equations for the ellipses are: $1=\frac{x^2}{36}+\frac{y^2}{9}$ (outer ellipse) $1=\frac{x^2}{25}+\frac{y^2}{4}$ (inner ellipse). Solve both equations for y. $y_1=\left(9-\frac{9x^2}{36}\right)^{1/2}$; $y_2=\left(4-\frac{4x^2}{25}\right)^{1/2}$ Now

$$2\left[\int_{-6}^{6}(y_1)dx-\int_{-5}^{5}(y_2)dx\right]=8\pi$$



Question # 15:

Give the cookie as a circle $x^2+y^2=36$. Make a vertical cut extending through the first and fourth quadrants. Make the area of to the right of the cut in the first quadrant 6π. Solve $\int_x^6 \sqrt{36-x^2}\,dx=6\pi$

Use a graphing calculator to find that $x=1.61$ Then $y=\pm 5.78$ so the cut goes from (1.61, -5.78) to (1.61, 5.78) so the total length is 11.56