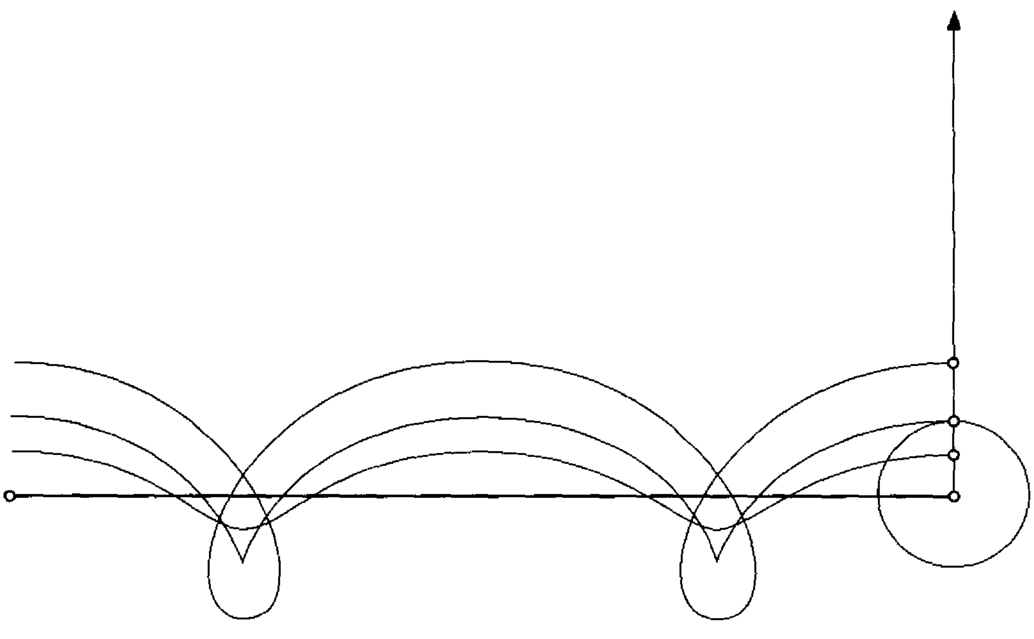
6

Circles



Introducing Circles

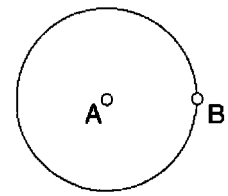
Name(s): ______________________

A circle is the set of all points in a plane the same distance from a given point. In this activity, you'll construct circles and investigate the meaning of this definition and other circle definitions.

Sketch and Investigate

The **Compass** tool

1. Use the **Compass** tool to construct a circle. Two points define your circle.



The **Selection Arrow** tool

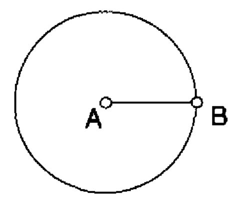
2. Choose the **Selection Arrow** tool and drag each point to see how it affects the circle.

Q1 Describe how each point affects the circle.

The **Text** tool

Click once on an object to show or hide its label. Drag the label to move it. Double-click the label to change it.

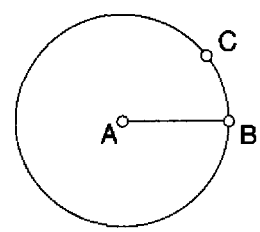
3. Choose the **Text** tool. Show the point labels and change them, if necessary, to match the diagram.
4. Choose the **Segment** tool and construct segment AB. This segment is called a *radius*.



5. Choose the **Selection Arrow** tool. Select the segment and, in the Measure menu, choose **Length**. This length is also called the *radius* of the circle.

In most books, a circle is named after its center point. The circle shown at right would be called circle *A*. However, it's often convenient to name a Sketchpad circle after both points that define it, so you could call the circle at right circle *AB*.

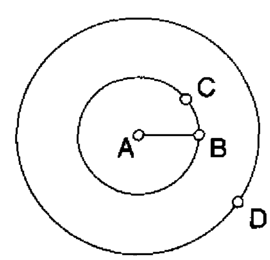
6. Select the circle and, in the Measure menu, choose **Radius**.
7. Drag point A or point B and observe these measures.
8. Use the **Point** tool to construct point C on the circle.



9. Choose the **Selection Arrow** tool. Click in blank space to deselect all objects. Select point A and point C. In the Measure menu, choose **Distance**.
10. Drag point C around the circle and observe the distance AC.

Q2 True or false: The radius of a circle is the distance from the center to *any* point on the circle. ______________

11. Construct a circle centered at point A with control point D, as shown at right.
12. Drag point D so that circle AD moves inside and outside circle AB.



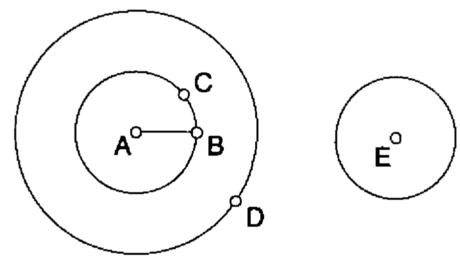
Q3 If two or more coplanar circles share the same center, they are *concentric* circles. How many circles can share the same center? (Why might it be convenient to name circles after two points?)

13. Construct a point E anywhere in your sketch.

Select $\overline{AB}$ and point E; then, in the Construct menu, choose **Circle By Center+Radius**.

14. Construct a circle with center E and radius AB.

15. Drag point B to see how it affects circle E.



Sketchpad will call the circle something like "circle c1" instead of "circle E."

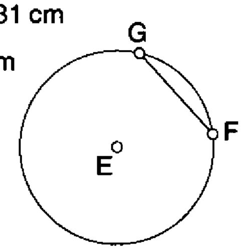
16. Measure the radius of circle E.

Q4 Circle E and circle AB are *congruent circles*. Write a definition of congruent circles.

17. Construct $\overline{FG}$, where points F and G are points on circle E. This segment is called a *chord* of the circle.

18. Measure the length of $\overline{FG}$.

19. Drag point F around the circle and observe the length measure.



Q5 Make the length of $\overline{FG}$ as great as you can. When a chord in a circle is as long as it can possibly be, it is called a *diameter*. Describe a diameter.

Q6 The length of a diameter segment is also called the diameter of the circle. How does the diameter of a circle compare to the radius?

Explore More

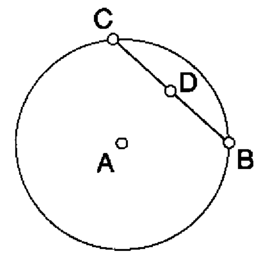
1. In the activity, you constructed a chord that you could make into a diameter by dragging one of its endpoints. But this chord won't stay a diameter if you change the circle. Figure out a way to construct a circle and a diameter that will always stay a diameter when you drag. (*Hint:* There are many ways to do this. One way uses a ray.) Describe your method.

Chords in a Circle

Name(s): ____________________

A chord in a circle is a segment with endpoints on the circle. In this activity, you'll investigate properties of chords.

Sketch and Investigate



1. Construct circle AB.
2. Construct chord BC.
3. Construct the midpoint D of the chord.
4. Construct a line through point D perpendicular to $\overline{BC}$. This line is the perpendicular bisector of the chord. (It's not shown in the figure.)

   Select point D and $\overline{BC}$; then, in the Construct menu, choose **Perpendicular Line**.

5. Drag point C around the circle and observe the perpendicular line.

Q1 Write a conjecture about the perpendicular bisector of any chord in a circle.

6. Hide the perpendicular bisector and construct $\overline{AD}$.

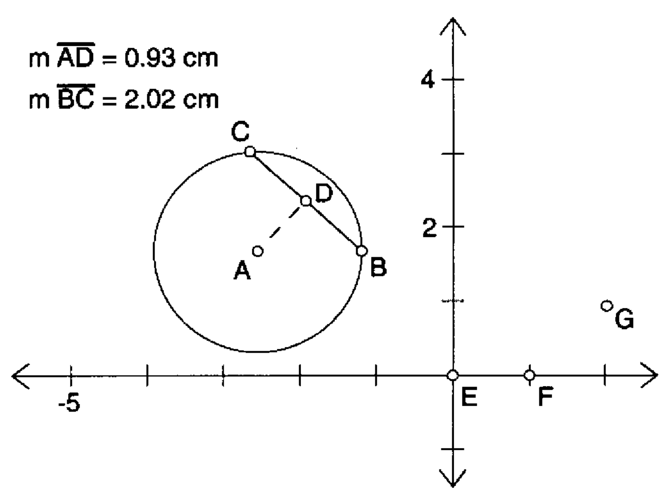
   Select the line; then, in the Display menu, choose **Hide Line**.

7. Measure the length of $\overline{AD}$. This is the distance from the chord to the center.
8. Measure the length of $\overline{BC}$.
9. Drag point C around the circle and observe the measures.

Q2 How is the length of the chord related to its distance from the center?

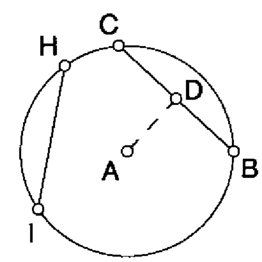
10. You can make a graph that shows this relationship: Select the length of $\overline{BC}$ and the length of $\overline{AD}$, in that order; then, in the Graph menu, choose **Plot As (x, y)**. You should get axes and a point G whose coordinates are the measures you selected.

    If you don't see point G, scale the axes by dragging point F toward point E.



11. Drag point C to see how it controls point G.
12. To graph all the possible locations for point G, select it and point C; then, in the Construct menu, choose **Locus**.
13. Drag point C to see point G travel along the locus.
14. Drag point A or point B to see what effect changing the circle's radius has on the graph.

Q3 Write a paragraph describing the graph. Answer these questions in your paragraph: Look at the value of y where the locus intersects the y-axis. What does this value represent in the circle? Look at the value of x where the locus intersects the x-axis. What does this value represent in the circle? As point G moves from left to right, what happens to the value of its y-coordinate? What does this have to do with what's happening to the chord? Use a separate sheet of paper.



15. Construct $\overline{HI}$, another chord on the circle.
16. Measure HI.
17. Measure the distance from $\overline{HI}$ to the center of the circle.

Select $\overline{HI}$ and point A; then, in the Measure menu, choose **Distance**.

18. Drag point H or point I and watch the length measure. Try to make this length as close to the length of $\overline{BC}$ as you can.

Q4 Write a conjecture about congruent chords in a circle.

Explore More

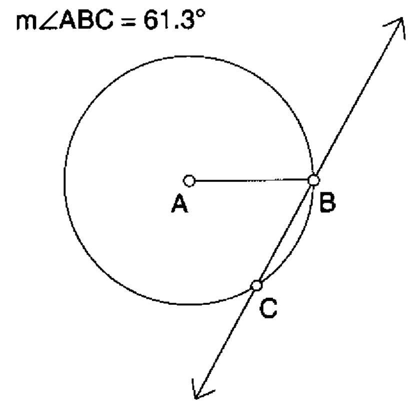
1. Plot as (x, y) the length of $\overline{HI}$ and the distance from $\overline{HI}$ to the center. How does this plotted point compare to point G when $HI = BC$?
2. An arc is part of a circle. You can construct an arc from any three points. In a new sketch, construct a three-point arc. Now use your conjecture from Q1 to construct the center of the circle containing the arc. Construct the circle to confirm that you found the correct point. Explain what you did.

Tangents to a Circle

Name(s): ____________________

A line can intersect a circle in zero, one, or two points. A line that intersects a circle in exactly one point—that just touches the circle without going into the circle's interior—is called a *tangent*. The point of intersection is called the *point of tangency*. A line that intersects a circle in two points is called a *secant*. In this investigation, you'll construct a secant, then manipulate it until it becomes a tangent to discover an important property of tangents.

Sketch and Investigate



1. Construct circle AB.
2. Construct $\overline{AB}$.
3. Construct secant $\overleftrightarrow{BC}$, making sure point C falls on the circle.
4. Measure $\angle ABC$.
5. Drag point C around the circle and observe the angle measure.

Hold down the mouse button on the **Segment** tool, then drag right to choose the **Line** tool.

Select, in order, points *A*, *B*, and *C*. Then, in the Measure menu, choose **Angle**.

Q1 What happens to m$\angle ABC$ as point C gets closer to point *B*? What's the measure of $\angle ABC$ when point C is right on top of point *B*?

Q2 When points *B* and *C* coincide, your line intersects the circle in a single point, so it's tangent to the circle. How is a tangent related to the radius at the point of tangency?

Q3 Use what you observed in Q2 to construct a line in your sketch that is always tangent to the circle. Describe how you did it.

Explore More

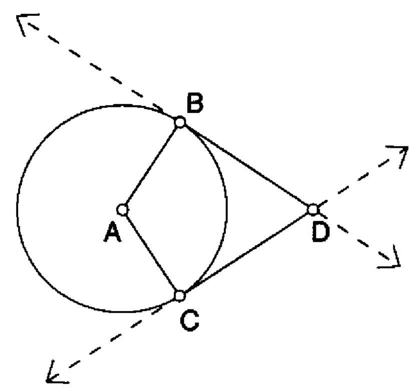
1. Come up with methods for constructing two circles that always intersect in one point. The circles could be *internally tangent* (one inside the other) or *externally tangent* (neither inside the other).

Tangent Segments

Name(s): ______________________

In this activity, you'll learn how to construct tangents. Then you'll compare the lengths of two tangent segments from the common intersection point to the points of tangency.

Sketch and Investigate



1. Construct circle AB and radius AB.

Select $\overline{AB}$ and point B; then, in the Construct menu, choose **Perpendicular Line**.

2. Construct a line perpendicular to $\overline{AB}$ through point B. This line is tangent to the circle.
3. Drag point B to confirm that the line stays tangent.
4. Construct a second radius AC.
5. Construct a tangent through point C.
6. Drag point C to confirm that this line stays tangent.
7. Construct point D where the tangent lines intersect.

Select the lines; then, in the Display menu, choose **Hide Lines**.

8. Hide the lines.
9. Construct segments BD and CD.
10. Measure BD and CD.
11. Drag point C and observe the measures.

Q1 Write a conjecture about tangent segments.

Explore More

1. Construct $\overline{AD}$. Investigate relationships among the angles and sides of the two triangles formed. Are the triangles congruent? If so, explain why.
2. Construct a circle and a point outside the circle. Come up with a method for constructing two tangents from the given point. Describe your method.
3. Come up with methods for constructing two or more circles with a common tangent (a line tangent to both circles). *Hint:* Construct the second circle after you've constructed the first circle's tangent. Describe your method.

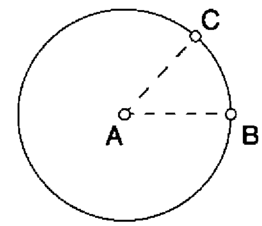
Arcs and Angles

Name(s): ______________________

An angle with its vertex at the center of a circle is called a *central angle*. An angle whose sides are chords of a circle and whose vertex is on the circle is called an *inscribed angle*. In this activity, you'll investigate relationships among central angles, inscribed angles, and the arcs they intercept.

Sketch and Investigate

1. Construct circle AB.



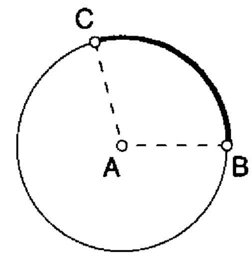
Select the segment and choose **Display | Line Width | Dashed**.

2. Construct $\overline{AB}$ and make this segment dashed.
3. Construct $\overline{AC}$, where point C is a point on the circle.

You've just created central angle BAC. Points B and C divide the circle into two arcs. The shorter arc is called a *minor arc* and the longer one is called a *major arc*. A minor arc is named after its endpoints. In the figure above right, the central angle BAC intercepts $\overset{\frown}{BC}$, where $\overset{\frown}{BC}$ is the minor arc.

Select, in order, point B, point C, and the circle. Then, in the Construct menu, choose **Arc On Circle**.

4. Construct the arc on the circle from point B to point C. While the arc is selected, make it thick.
5. Drag point C around the circle to see how it controls the arc. When you're finished experimenting, locate point C so that the thick arc is a minor arc.



Select $\overset{\frown}{BC}$; then, in the Measure menu, choose **Arc Angle**.

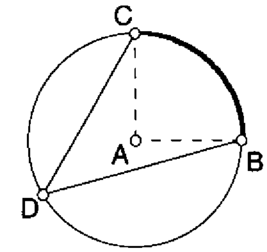
6. Measure the arc angle of $\overset{\frown}{BC}$.

Select, in order, points B, A, and C. Then, in the Measure menu, choose **Angle**.

7. Measure $\angle BAC$.
8. Drag point C around the circle again and observe the measures. Pay attention to the differences when the arc is a minor arc and when it is a major arc.

Q1 Write a conjecture about the measure of the central angle and the measure of the minor arc it intercepts.

Q2 Write a conjecture about the measure of the central angle and the measure of the major arc.



9. Construct $\overline{DC}$ and $\overline{DB}$, where point D is a point on the circle, to create inscribed angle CDB.
10. Measure $\angle CDB$.
11. Drag point C and observe the measures of the arc angle and $\angle CDB$.

Q3 Write a conjecture about the measures of an inscribed angle and the arc it intercepts.

12. Drag point D (but not past point C or point B) and observe the measure of $\angle CDB$.

Q4 Write a conjecture about all the inscribed angles that intercept the same arc.

13. Drag point C so that the thick arc is as close to being a semicircle as you can make it.
14. Drag point D and observe the measure of $\angle CDB$.

Q5 Write a conjecture about angles inscribed in a semicircle.

Explore More

1. In a new sketch, construct a circle and an arc on the circle. Measure the circumference of the circle, the arc angle, and the arc length. Use the circumference and arc angle measurements to calculate an expression equal to the arc length. Explain what you did.
2. Use your conjecture in Q5 to come up with a method for constructing a right triangle. Describe your method.

The Circumference/Diameter Ratio

Name(s): ______________

In this activity, you'll discover a relationship between a circle's circumference and its diameter. Even if that relationship is familiar to you, the investigation may demonstrate it in a different way.

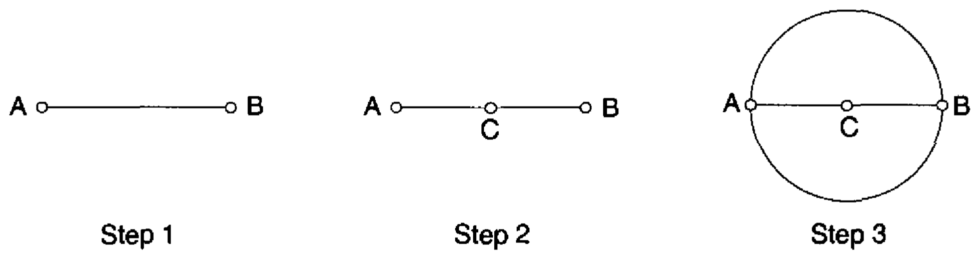
Sketch and Investigate

1. Construct $\overline{AB}$.

2. Construct point C, the midpoint of $\overline{AB}$.

Be sure the cursor is positioned directly on point B when you release the mouse button.

3. Construct circle CB.



Select the circle; then, in the Measure menu, choose **Circumference**.

4. Measure the circumference of the circle.

5. Measure AB (the diameter of the circle).

6. Make the circle small.

Select, in order, the length measurement and the circumference measurement. Then, in the Graph menu, choose **Tabulate**.

7. Make a table for the length measurement and the circumference measurement.

Double-click inside the table to add an entry.

8. Make the circle a little bigger; then add an entry to the table.

9. Repeat step 8 until your table has at least four entries.

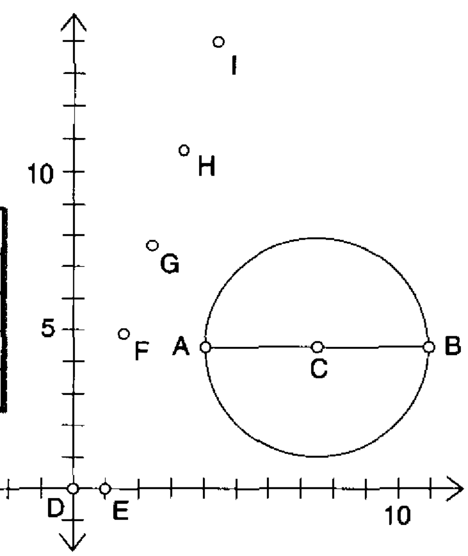
Choose **Plot Points** from the Graph menu and enter the coordinates of the points in your table.

10. Plot the table data. You may need to drag point E toward point D to scale your axes so that you can see the points.

m $\overline{AB}$ = 2.999 cm

Circumference ⊙CB = 9.420 cm

m $\overline{AB}$	Circumference ⊙CB
1.552 cm	4.876 cm
2.434 cm	7.647 cm
3.387 cm	10.640 cm
4.445 cm	13.964 cm



Q1 Describe the points that appear on the graph.

Select, in order, the diameter measurement and the circumference measurement. Then, in the Graph menu, choose **Plot As (x, y)**.

11. Plot the diameter and circumference measurements as (x, y). Change the color of this point so you can tell it from other points on the graph. Also, in the Display menu, choose **Trace Point**.
12. Drag point A or point B to change your circle. Watch the plotted point.
13. Construct a ray from point D (the origin) to any of the plotted points.
14. Measure the slope of the ray.

Q2 How is the slope of the ray related to circumference/diameter ratio for the circles?

Q3 What's the significance of the fact that all the plotted points lie on this ray?

Q4 The circumference/diameter ratio is represented by the Greek letter π (pi). Complete the following formulas using π, C for circumference, and D for diameter:

π = __________

C = __________

Q5 Write a formula for circumference using C, π, and r (for radius).

The Cycloid

Name(s): ____________________

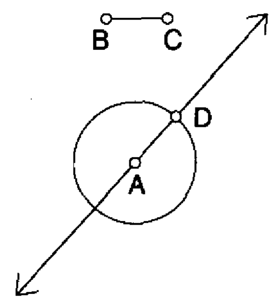
Imagine that a bug is clinging tightly to a bicycle wheel as the bicycle travels down the road. What path does the bug travel? Is the path the same whether the bug is at the center of the wheel or at its edge? In this activity, you'll investigate those questions.

Sketch and Investigate

You'll start by constructing a stationary circle with a rotating spoke.

Select the point and the segment; then, in the Construct menu, choose **Circle By Center+Radius**.

1. Draw a point A and a short segment BC.
2. Construct the circle centered at A with radius equal to BC.
3. Drag point B or C and note how segment BC controls the radius of the circle.



Select point D; then, in the Display menu, choose **Animate Point**. To stop the animation, choose **Stop Animation** or press the Stop button on the Motion Controller.

4. Construct line $\overleftrightarrow{AD}$, where D is a point on the circle.
5. Animate point D around the circle.

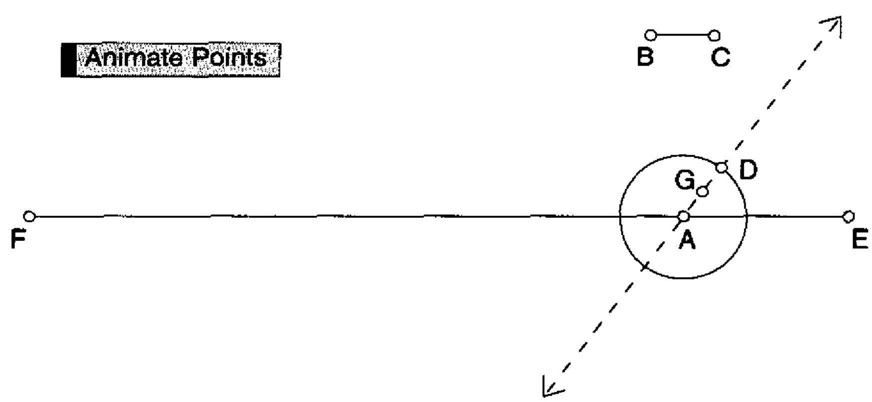
Now you'll attach the circle to a straight path and construct a point representing the bug.

To make it easy to draw a horizontal segment, hold down the Shift key while you draw the segment.

6. Construct a long horizontal segment EF from right to left.
7. Merge point A onto the segment. To do this, select point A and $\overline{EF}$ and choose **Merge Point To Segment** from the Edit menu.
8. Construct point G on line AD, inside the circle. (Point G represents the bug.)
9. While point G is selected, choose **Trace Point** in the Display menu.
10. Hide line AD.

Select, in order, points A and D; then choose **Edit | Action Buttons | Animation**. Choose **forward** from the Direction pop-up menu, then click OK.

11. Make an action button to animate point A forward at medium speed and point D counter-clockwise at medium speed.



The Cycloid (continued)

If the traces clutter the screen, choose **Erase Traces** from the Display menu. Or you can choose **Preferences** from the Edit menu, go to the Color panel, and check the Fade Traces Over Time box.

12. Press this new action button to observe the path of point *G*. This path is called a **cycloid**. (Press the button again when you wish to stop the animation.)

13. Try the animation with point *G* on the circle and again with point *G* outside the circle.

Q1 In the space below, sketch what the cycloid looks like when point *G* is inside the circle, on the circle, and outside the circle.

Inside the circle	On the circle	Outside the circle

14. Measure the length of segment *EF* and the circumference of the circle.

15. Adjust your sketch so that point *G* traces exactly two cycles of the curve. Note the two measurements. Then adjust the sketch so that point *G* traces three cycles.

Q2 How are the circumference of the circle and the length of the segment related when point *G* traces two cycles? three cycles?

Q3 Because the cycloid curve repeats itself, it is called *periodic*. The distance from a point on one cycle to the corresponding point on the next cycle (for example, the distance from a peak to a peak) is called the *period* of the curve. What would be the period of the curve if the circle had a radius of 1 cm?

Q4 How does point *G*'s position—inside, outside, or on the circle—affect the period of the cycloid?