

I use the following mnemonic devices to help students remember the steps required in carrying out a hypothesis test or confidence interval: **PHANTOMS** and **PANIC**.

**P** arameter  
**H** ypotheses  
**A** ssumptions  
**N** ame the test  
**T** est statistic  
**O** btain p-value  
**M** ake decision  
**S** tate conclusion in context

**P** arameter  
**A** ssumptions  
**N** ame the interval  
**I** nterval  
**C** onclusion in context

Example:

A study of iron deficiency in infants compared samples of infants whose mothers chose different ways of feeding them. One group contained breast-fed infants. The children in another group were fed a standard baby formula without any iron supplements. Here are summary results on blood hemoglobin levels at 12 months of age.

| Group      | n  | $\bar{x}$ | s   |
|------------|----|-----------|-----|
| Breast-Fed | 23 | 13.3      | 1.7 |
| Formula    | 19 | 12.4      | 1.8 |

Is there significant evidence that the mean hemoglobin level is different among breast-fed babies? Give a 95% confidence interval for the mean difference in hemoglobin level between the two populations of infants.

### **HYPOTHESIS TEST:**

**P**  $\mu_{breast}$  : the mean blood hemoglobin level at 12 months of age for breast-fed babies  
 $\mu_{formula}$  : the mean blood hemoglobin level at 12 months of age for formula-fed babies

**H**  $H_0 : \mu_{breast} = \mu_{formula}$   
 $H_0 : \mu_{breast} \neq \mu_{formula}$

**A** It is reasonable to assume that the samples were chosen randomly and independently. It is also reasonable to assume that at least 230 breast-fed babies and 190 formula-fed babies are present in the population. Although we cannot check for outliers or strong skewness, the combined sample size of 42 is sufficiently large.

**N** Therefore, a two-sample t-test may be used.

$$t = \frac{(\bar{x}_{breast} - \bar{x}_{formula}) - (\mu_{breast} - \mu_{formula})}{\sqrt{\frac{S^2_{breast}}{n_{breast}} + \frac{S^2_{formula}}{n_{formula}}}} = \frac{(13.3 - 12.4) - (0)}{\sqrt{\frac{1.7^2}{23} + \frac{1.8^2}{19}}} \approx 1.6537$$

$$df \approx 37.5976 \quad p = 2P(t \geq 1.6537) \approx 2(0.0533) \approx 0.1065$$
$$tcdf(1.6537, 1E99, 37.5976) \approx 0.0533$$

**M** Do not reject  $H_0$ .

**S** There is not strong enough evidence to conclude that the mean blood hemoglobin level at 12 months of age for the breast-fed group is significantly different than that of the formula-fed group. If there truly is no difference between breast-fed and formula-fed babies, we would expect a result at least this extreme in about 10 out of every 100 samples due to chance.

#### CONFIDENCE INTERVAL:

**P**  $\mu$ : the mean difference in blood hemoglobin level at 12 months of age between breast-fed babies and the formula-fed babies

**A** It is reasonable to assume that the samples were chosen randomly and independently. It is also reasonable to assume that at least 230 breast-fed babies and 190 formula-fed babies are present in the population. Although we cannot check for outliers or strong skewness, the combined sample size of 42 is sufficiently large.

**N** Therefore, a two-sample t-interval may be used.

$$(\bar{x}_{breast} - \bar{x}_{formula}) \pm t * \sqrt{\frac{S^2_{breast}}{n_{breast}} + \frac{S^2_{formula}}{n_{formula}}} = (13.3 - 12.4) \pm 2.021 \sqrt{\frac{1.7^2}{23} + \frac{1.8^2}{19}} = 0.9 \pm 1.0999$$

**I** (-0.2021, 2.0021)

**C** We are 95% confident that the true mean level of hemoglobin at 12 months of age in breast-fed babies is between about 0.2 units lower and 2.0 units higher than that of formula-fed babies, since 95% of all samples of this size would produce a mean difference within 1.10 units of the true mean difference.