

Middletown

January Regional 1/10/04

Precalculus Team Question #1

For all real nonzero numbers, $f(x) = 1 - \frac{1}{x}$ and $g(x) = 1 - x$. If $h(x) = f[g(x)]$, for what value of x does $h(x) = 8$?

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Precalculus Team Question #2

In acute triangle ABC , $AB = c$, $BC = a$, and $CA = b$, and $ac = 2b$. Find the numerical value of $\frac{\cos A}{a} + \frac{\cos C}{c}$.

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Precalculus Team Question #3

Solve the inequality $\frac{1}{|x-1|} \geq 3$.

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Precalculus Team Question #4

The cost for one print run of a book is jointly proportional to the number of pages in the book and the number of books in the print run. It costs \$20,000 to print 4000 copies of a 100-page book. Find the cost to print 400 copies of a 293-page book.

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Precalculus Team Question #5

The Bay of Fundy has the highest tides in the world. In one 12-hour period, the water starts at mean sea level, rises to 21 feet above, drops to 21 feet below, then returns to mean sea level. Assuming the motion of the tide is simple harmonic, find an equation, in terms of sine, that describes the height of the tide in the Bay of Fundy above mean sea level.

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Precalculus Team Question #6

Find the exact value of $\sin \frac{\pi}{100} + \sin \frac{2\pi}{100} + \sin \frac{3\pi}{100} + \dots + \sin \frac{199\pi}{100}$ where the sum contains the sines of all numbers of the form $\frac{k\pi}{100}$, with k varying from 1 to 199, and k is a natural number.

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Precalculus Team Question #7

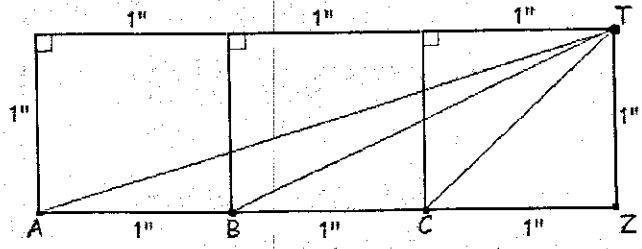
If $0 < \theta < \frac{\pi}{2}$ and $\sin 2\theta = a$, find $\sin \theta + \cos \theta$ in terms of a .

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Precalculus Team Question #8

Find the exact sum

$(m\angle TAZ + m\angle TBZ + m\angle TCZ)$



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Precalculus Team Question #9

A housing contractor has subdivided a farm into 100 building lots. He has designed 2 types of homes for these lots: colonial and ranch style. A colonial requires \$30,000 of capital and produces a profit of \$4000 when sold. A ranch style house requires \$40,000 of capital to build and provides an \$8000 profit when sold. If he has \$3.6 million of capital, how many houses of each type should he build for maximum profit?

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Precalculus Team Question #10

Suppose a baseball is thrown at 30 fps at a 60-degree angle to the horizontal, from a height of 4 feet above the ground.

Let A = the horizontal distance, in feet, that the ball has traveled (to the nearest hundredth)

Let B = the time, in seconds, of when the baseball hits the ground (to the nearest hundredth).

Find the sum of $A + B$.

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Precalculus Team Question #11

Given $\sin B = \frac{a^2 - b^2}{a^2 + b^2}$ and $\cos B = \frac{2ab}{a^2 + b^2}$. Find: $\tan \frac{B}{2}$, if angle B is in the first quadrant.

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Precalculus Team Question #12

On a special flight, there are 9 boys, 5 American children, 9 men, 14 Americans, 6 American males, and 7 foreign females. Find the minimum number of people that could be on the plane.

(For this problem, boys are under 18 and men are over 18).

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Precalculus Team Question #13

In $\triangle ABC$, the ratio of side BC to side AC is $\sqrt{10} : \sqrt{15}$. If $A = \text{Arc tan } 1$, find the exact size of $\angle C$ in radians.

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Precalculus Team Question #14

If $i = \sqrt{-1}$, find the ordered pair of real numbers (a, b) for which $a + 4i$ and $b + 5i$ are the roots of $x^2 - (10 + 9i)x + (4 + 46i) = 0$.