1. In the box shown, each dimension l, w and h, is a positive integer greater than 1. The area of the base (shaded) is 15 sq. cm and the area of one of the faces is 20 sq. cm. If the volume of the box is K and the surface area of the box is S then give the value of K+S.


#1

May not be drawn to scale.
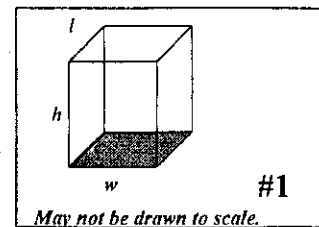
2. The radius of the circle with center A is three times that of the radius of the circle with center B. The degree measure of $\angle CAD$ is half that of the degree measure of $\angle EBF$. The area of the shaded sector of circle A is 10. What is the area of the shaded sector of circle B?
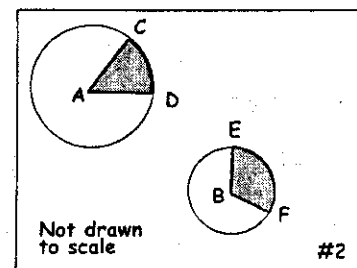
3. Add 6 to $\cos x$. Divide this sum by 4. Subtract 2 from this quotient. The result is -0.3115. For $0 < x < 2\pi$, give the greatest possible value of x, to the nearest tenth place.

Not drawn to scale

#2

4. If the area of $\triangle ABC$ is 20, and $\angle B$ is obtuse, give the value of $\cos B$.

5. $\triangle ABC \sim \triangle DEF$ and the area of $\triangle ABC$ is $\frac{3}{2}$ times the area of $\triangle DEF$. If $AC = \tan\theta$ and $DF = \sin\theta$ for $0 < \theta < 90°$ then give the value of θ to the nearest tenth of a degree.

#4
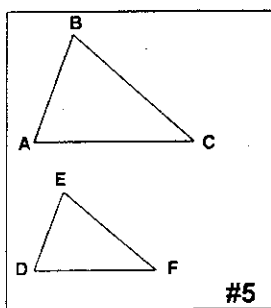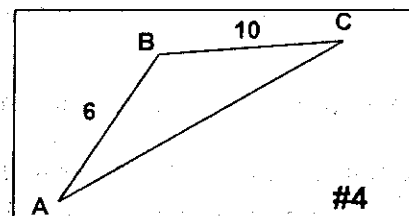
6. Consider circle A with equation
$$x^2 + y^2 + 2x + 14y - 50 = 0.$$
Let the area of circle A be $B\pi$.
Let the center of circle A be the point (C, D). Let the maximum y-value on the graph of circle A be E. Give the value of $B + C + D + E$.

#5

7. $\sum\limits_{n=10}^{50} (4) = k$. A circle of radius k is inscribed in a square. Give the length of the diagonal of the square.

8. For $f(x) = \frac{1}{x+1}$ $f(f(f(5))) = \frac{A}{B}$, for A and B relatively prime integers.
$f(f(\frac{C}{D})) = 6$ for C and D relatively prime integers. Give the value of $|A| + |B| + |C| + |D|$.

9. A number, written in base five, $231X_{five}$ is equal to the number $3X3_{ten}$ written in base ten. (X is a digit in each number.) A number in base seven $2A3_{seven}$ is equal to 129_{ten} in base ten (A is a digit in each number.) Give the value of $X + A$.

10. Let $C(n, r)$ denote the number of possible combinations of n objects, taken r at a time. If $C(n, 3) = 20n - 20$ for $n > 3$, then give the value of n.

11. Let $4^{1-x} = \frac{1}{64}$ and $2^{\sqrt{3y}} = 8$. In $\triangle ADC$, D is a right angle, and $\overline{DB}$ is the altitude to its hypotenuse. If $AD = x$ and $DC = y$ then find the length AB.

12. A triangle has sides of length $3 + m$, $5 + m$, and $m + 1$, and a perimeter of 45. A convex hexagon has angles of measures $120°, 130°, 100°, 2n°, 2n°$ and $n°$. The graph of $3x - 2y = 12$ has a slope of $\frac{6(n+m)}{k}$. Give the value of k.

13. The numbers a, b and c have an arithmetic mean of 40. If b and c are each increased by $2\cos\frac{\pi}{4}$ and a is decreased by $\sec^2\left(\frac{\pi}{4}\right)$, give the arithmetic mean of the three new values.

14. $4x^2 + 9y^2 = 36$ has rectangle ABCD inscribed as shown. The center of ABCD is on the origin. The length (AB) of ABCD is 2. Give the area of ABCD.
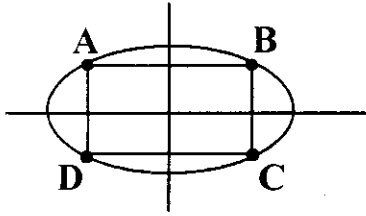
15. Let $f(x) = \frac{1}{x+1}$ and $g(x) = \frac{x+6}{x-2}$. The domain of $f(g(x))$ is $(-\infty, A) \cup (A, B) \cup (B, +\infty)$.

Give the value of $\frac{A}{B}$.