

Remember that **NOTA** means None of the Above.

1. The equation $y^3 = 3x^3 + x^7$ is symmetric about

- a) the x -axis b) the y -axis
c) the origin d) There is no symmetry
e) **NOTA**

2. How many real values of x satisfy the equation $\tan[\text{Arc tan } x + \text{Arc tan } x^2] = x$?

- a) 0 b) 1 c) 2 d) No real solns. E) **NOTA**

3. If $0 < x < \pi$ and $2^{\tan x} = 8^{\sin x}$, Which value of $\cos(x)$ makes the equation true?

- a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$ e) **NOTA**

4. Suppose $x + y = 1$ and $x^2 + y^2 = 4$, then find $x^3 + y^3$.

- a) $\frac{5}{2}$ b) 4 c) $\frac{11}{2}$ d) 8 e) **NOTA**

5. The complex number $a + bi$, when squared, gives $-3 + 4i$. If $a > 0$, find the ordered pair (a, b) .

- a) (1, 2) b) (2, 1) c) (1, 3) d) (3, 1) e) **NOTA**

6. Solve for all real value(s) of x :

$$2^{41} = 2 + \sum_{k=0}^{39} \log_{10} x^{2^k}$$

- a) 2 b) 10 c) 100 d) 200 e) **NOTA**

7. Solve: $|5x - 2| < 6$

a) $x < \frac{-4}{5} \cup x > \frac{8}{5}$ b) $x < \frac{8}{5}$

c) $x > \frac{-4}{5}$ d) $\frac{-4}{5} < x < \frac{8}{5}$ e) **NOTA**

8. If the fraction $\frac{2}{(x-1)(x^2+x-4)}$ is resolved into partial fractions, there will be two constant terms in the new numerators. What is the sum of those constant terms?

- a) 0 b) 1 c) 2 d) 4 e) **NOTA**

9. One bag contains 3 white and 2 black balls. A 2nd bag contains 2 white and 3 black balls. A ball is drawn from the 2nd bag and placed in the 1st bag. Then a ball is drawn from the 1st bag and placed into 2nd bag. When the pair of operations is repeated, what is the probability that the 1st bag will contain 5 white balls?

- a) $\frac{2}{41}$ b) $\frac{1}{115}$ c) $\frac{1}{225}$ d) $\frac{1}{297}$ e) **NOTA**

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10. The cable of a suspension bridge has supporting towers that are 50 feet high and 400 feet apart. If the lowest point of the cable is 10 feet above the floor of the bridge, find the height of a supporting rod 100 feet from the center of the span (in feet).

The cable lies on a parabolic curve.

- a) 10 b) 15 c) 20 d) 25 e) NOTA

11. Given the equation of the hyperbola $9x^2 - 4y^2 - 36x + 32y + 8 = 0$. Find the length of the latus rectum.

- a) $\frac{\sqrt{13}}{3}$ b) $\frac{8}{3}$ c) $\frac{18}{3}$ d) $\frac{\sqrt{13}}{4}$ e) NOTA

12. The following parametric equations represent the graph of an ellipse. $\cos \theta = \frac{x}{3}$ and $\sin \theta = \frac{y}{4}$. Find the length of the minor axis of this ellipse.

- a) 3 b) 4 c) 6 d) 8 e) NOTA

13. A 10-inch radius lawn roller makes 1.2 revolutions per second. Find the exact angular speed of the roller in radians per second.

- a) 1.2π b) 2.4π c) 4.8π d) 6π e) NOTA

14. Using the same lawn roller as in the previous problem, find the linear speed of the tractor pulling it (to the nearest tenth) in inches per second.

- a) 37.7 b) 75.4 c) 150.8 d) 188.5 e) NOTA

15. Using the following 5 statements, and if a true statement = 1 and a false statement = 0, find the sum of the trues and falses of these statements.

1. $\log_b b^{2x} = 2x$ 2. $e^{x-1} = \frac{e^x}{e}$
 3. $\ln(x+y) = \ln x + \ln y$ 4. $\ln(x+y) = \ln(xy)$
 5. $\log_{10} \left(\frac{10}{x} \right) = 1 - \log_{10} x$

- a) 1 b) 2 c) 3 d) 4 e) NOTA

16. Find the component form of a vector that represents the velocity of an airplane descending at a speed of 100 mph at an angle of 30 degrees below the horizontal.

- a) $\langle -50\sqrt{3}, -50 \rangle$ b) $\langle -50, -50\sqrt{3} \rangle$
 c) $\langle -100\sqrt{3}, -100 \rangle$ d) $\langle -100, -100\sqrt{3} \rangle$
 e) NOTA

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17. Given the equation of the ellipse $2x^2 + y^2 = 7$, find the sum of the lengths of the major and minor axes.

- a) $2\sqrt{7}$ b) $2\sqrt{14}$ c) $4\sqrt{7}$
 d) $4\sqrt{14}$ e) NOTA

18. Which of the following best represents the graph of the equation $x^2 + 4x - 7y^2 + 70y = 171$?

- a) ellipse b) parabola c) hyperbola
 d) degenerate conic e) NOTA

19. The frog population in a pond grows continuously and exponentially. The current population is 198 frogs, and the annual growth constant is 0.16. Find the number of years needed for the frog population to reach 500 (to the nearest hundredth).

- a) 3.22 b) 5.79 c) 6.81 d) 7.24 e) NOTA

20. Given that $u = 3i + 2j$ and $v = -2j$. Find the sum of $|v| + |u+v|$.

- a) 3 b) 5 c) 7 d) 9 e) NOTA

21. Solve completely: $\frac{2x}{x-1} > 3$.

- a) $x > 1$ b) $x < 3$ c) $x > 2$
 d) $x > \frac{5}{2}$ e) NOTA

22. On a tiny remote island, where the death penalty still exists, a person can be granted mercy after receiving a death sentence in the following way: He is given 18 white balls and 6 black balls. He must divide them among 3 boxes with at least one ball in each box. Then blindfolded, he must choose a box at random, and then choose a ball from within this box. He arranges the balls in the boxes in order to maximize his chances of getting mercy. He receives mercy only if the chosen ball is white. The probability of the man receiving mercy is

- a) $\frac{11}{12}$ b) $\frac{3}{4}$ c) $\frac{10}{11}$ d) $\frac{8}{11}$ e) NOTA

23. For all real values of x for which the terms are defined, $\cot \frac{1}{4}x - \cot x = \frac{\sin kx}{(\sin \frac{1}{4}x)(\sin x)}$. The value of k is

- a) $\frac{3}{8}$ b) $\frac{5}{8}$ c) $\frac{3}{4}$ d) $\frac{5}{4}$ e) NOTA

24. A certain substance doubles its volume every minute. At 9 AM a small amount is placed in a container and at 10 AM the container just fills. The time at which the container was $\frac{1}{4}$ full was

- a) 9:15 AM b) 9:30 AM c) 9:45 AM
d) 9:50 AM e) NOTA

25. During a particular morning, a light signal goes on at precisely 9 AM. After that it goes on and off at equal intervals, each lasting a whole number of minutes. Later that morning it is observed that the light is off at 9:09, on at 9:17 and on at 9:58. The light will be on during that morning at both

- a) 10:30 & 11:21 b) 10:14 & 11:00
c) 10:23 & 11:01 d) 10:25 & 11:33 e) NOTA

26. During a series of challenge races between Mrs. Doker and her eldest son Jeff, each swam 2 laps of a regular swimming pool. It was observed when Jeff swam faster than his mother over the 2nd lap, his mother actually had led at the end of the 1st lap. When Jeff had led at the end of the 1st lap, his mother actually swam faster than him in the 2nd lap. There were 9 races in which Jeff swam faster than his mother on at least one lap. Mrs. Doker swam a faster time than Jeff on 7 first laps and 6 second laps. The minimum number of such challenge races was

- a) 9 b) 10 c) 11 d) 12 e) NOTA

27. In how many different ways can a careless office worker place 4 letters in 4 envelopes so that no one gets the right letter?

- a) 4 b) 9 c) 12 d) 16 e) NOTA

28. Solve for all real x : $e^{2x+6} - 4e^{x+6} - 5e^6 = 0$.

- a) $\ln 5$ only b) $\ln 1$ only c) $\ln 1$ & $\ln 5$
d) null set e) NOTA

29. Using the graph of the function $f(x) = \frac{x+10}{|x|+2}$.

Find the sum of the x -intercepts and y -intercepts of every asymptote that is horizontal or vertical.

- a) 0 b) 1 c) 2 d) 4 e) NOTA

30. A certain parabola passes thru the points (5, 1) and (13, -7) and has the y -axis as its directrix. What are the coordinates of all points at which the vertex of this parabola could be located?

- a) $(\frac{1}{2}, -2)$ & $(1, 2)$ b) $(4, 5)$ & $(0, -2)$
c) $(\frac{1}{2}, -2)$ & $(0, -2)$ d) $(4, 5)$ & $(\frac{1}{2}, -2)$
e) NOTA