Geometry Team Questions: Sponsor Copy
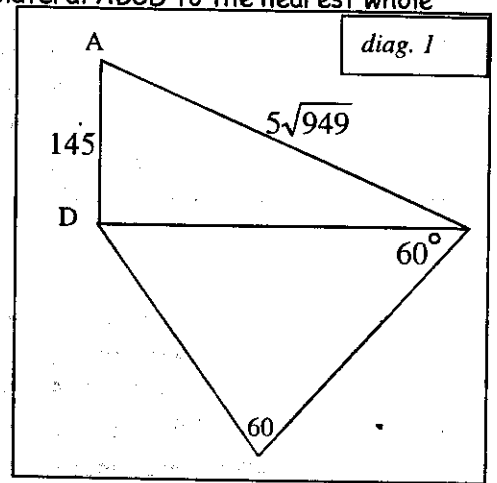
1. Let K be the length BC of quadrilateral ABCD. Let R be the area of quadrilateral ABCD to the nearest whole number. Give the value of $K^2 + R$. (diagram 1)

2. One exterior angle of a regular polygon is $4x$ degrees. One interior angle of the same polygon is $120x + 56$ degrees. If the polygon has sides of length 30 cm, then the perimeter of the polygon is P. Give the value of $\left(\sqrt{P}\right)^x$. (no diagram)

3. In $\triangle MST$, $\overline{RU}$ is parallel to $\overline{ST}$. If RU= $5x$, ST=8, MR= $x + 6$, and $RS = 4 - x$. If RU= y, then give the value of $14(x + y)$. (diagram 3)

4.

not drawn to scale.
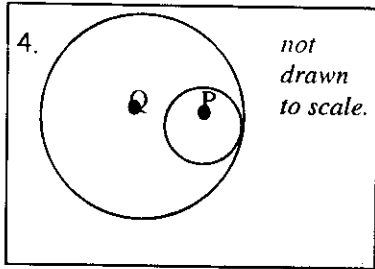
A circle with center P is internally tangent to a circle with center Q. The radius of P is 5 and the radius of Q is 20. If R is a point on circle P, and S is a point on circle Q, then let x be the greatest distance possible from R to S, and let y be the least distance possible between R and S, and z be length PQ. Give the value of $x + y + z$. (diag. 4)

diag. 1

A

$5\sqrt{949}$

145

D

$60°$

60

5. Consider triangles MAE, TAE, and ETH shown. MA is half of ME, and AT is half of AE. If EH = 45 and TH = 24 then give the perimeter of pentagon MATHE. (diagram 5)
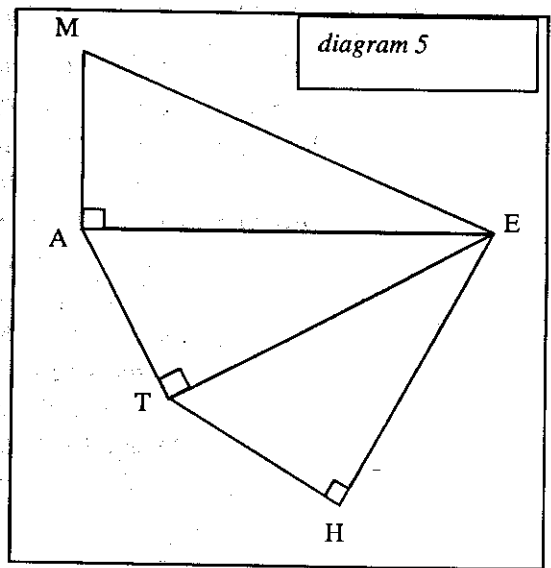
6. An equilateral triangle PQR is drawn on the coordinate plane, with vertices all equidistant from the origin. Each side of the triangle has length 12. Square ABCD is drawn on the same axes, with center on the origin, and vertices A(1, 1), B(1, -1), C(-1, -1), and D(-1, 1). Both polygons rotate about the origin, each vertex making a 360 degree rotation. Let K be the total area covered by the triangle during its rotation. Let L be the total area covered by the square during its rotation. Give the positive difference of K and L. (no diagram)

diagram 5

M

A

E

T

H

7.

E   D

I

F   J   G   H   C

A   B

Equilateral triangle AGB, regular pentagon ABHIJ, and regular hexagon ABCDEF all share a side $\overline{AB}$ as shown.
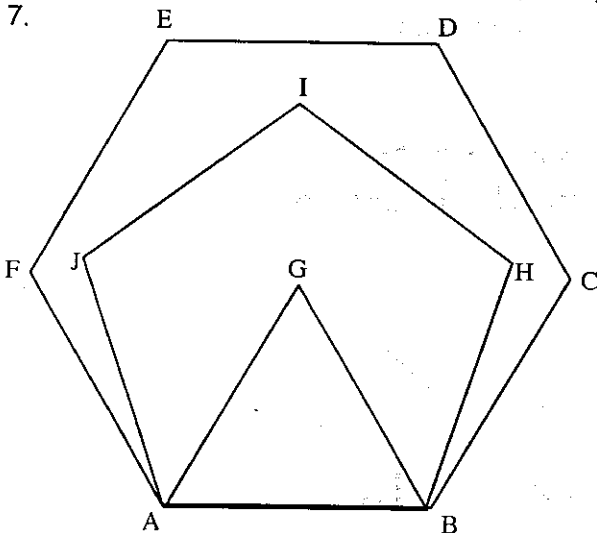
$x = m\angle JAG$
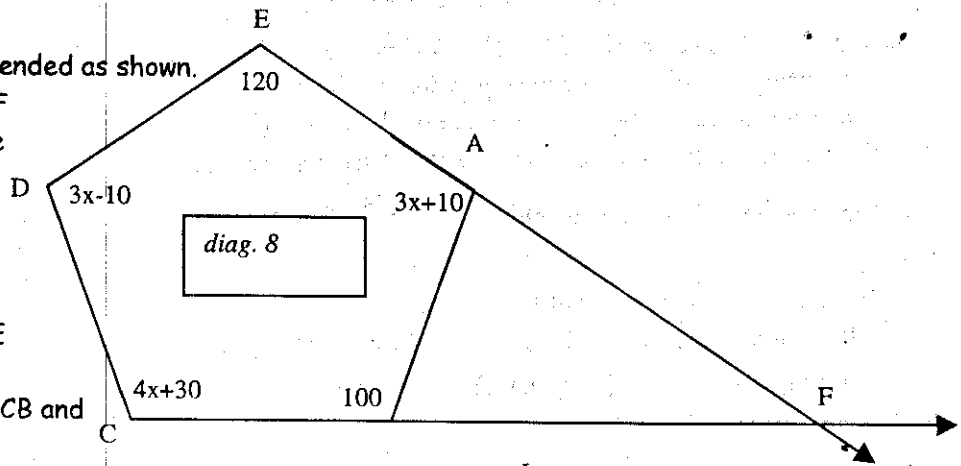
$y = m\angle FAJ$

$z = m\angle HGB$ (not shown)

$a = m\angle BCH$ (not shown)

Find the value of $x + y + z - a$.

8. Pentagon DEABC has two sides extended as shown. E, A and F are collinear, and C, B and F are collinear. If all expressions in the diagram represent angle measures, then give the measure of $\angle AFB$.

E

120

A

D

3x-10

3x+10

diag. 8

4x+30   100   F

C

9. In the figure shown, vertices L, H, E and B form a square with area 25. Congruent rectangles JIHL, GHFE, EDCB and ABLM, with square LHEB have areas that do not overlap, and together form the area of the entire figure. If the rectangles each have area 15, then give the distance from point J to point E.
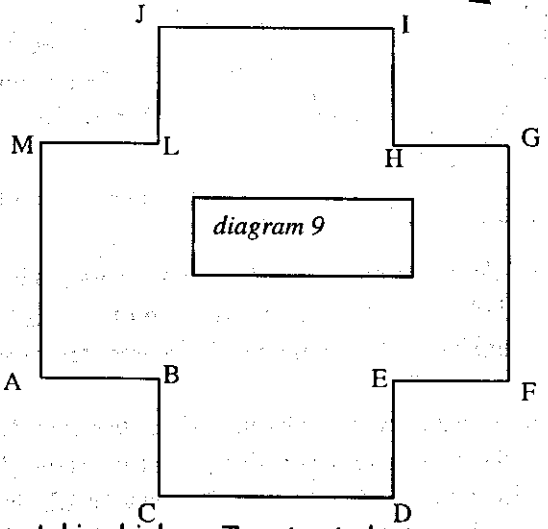
10. A rectangle ABCD has area R square cm. R is increased by 30%, giving result P. Rectangle EFGH has area P sq. cm. P is then decreased by 30% and the result is Q. Rectangle IJKL has area Q sq. cm. If rectangle ABCD has sides 12 cm and 8 cm and JK=10 cm, then give the length JL to the hundredth.

J   I

M   L   H   G

diagram 9

A   B   E   F

C   D

11. $m\angle C = 30^\circ$ and $\triangle ADB$ is an isosceles right triangle.

C
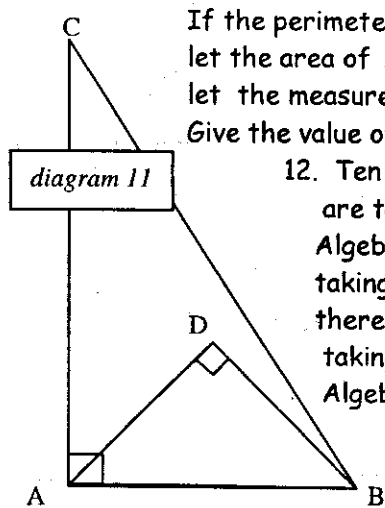
If the perimeter of $\triangle ADB$ is $12 + 6\sqrt{2}$ cm then let the area of $\triangle CAB$ be r square cm, let the measure of $\angle DBC$ be d degrees. Give the value of d + r, to the nearest tenth place.

diagram 11

D

A   B

12. Ten students at Countyview High School are taking biology. Twenty students are taking Algebra. Twelve students are taking Chemistry. Two students are taking Algebra and Chemistry. One student is taking Chemistry and Biology. Five students are taking Algebra and Biology. If one student is taking all three of these courses, and there are 50 students taking none of these courses then let x be the number of students taking Algebra but no biology, let y be the number of students taking Chemistry but no Algebra, let z be the total number of students at Countryview High School.

Give the value of $z \cdot \sqrt{x + y}$. (no diagram)

13. In parallelogram ABCD, the height to $\overline{CD}$ is $\overline{AE}$. If $m\angle ADC = 60^\circ$, and AD=10 and AB= 16 then let x be the height from B to side $\overline{AD}$. Let y be the length of the diagonal $\overline{AC}$. Let z be the area of ABCD.

Give the value of $\dfrac{100xy}{z}$ (no diagram).

14. In triangle ABD, $\overline{BD}$ is drawn, forming trapezoid BDEC, as shown. If BD=10, BC=15, DE=13, and CE=24 and the height of the trapezoid is 12. Find the area of triangle ABD.

A

B   10   D

15   12

C   E

13
24