

$$1. \quad A = \lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + 1}$$

$$B = \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x}$$

$$C = \lim_{x \rightarrow \infty} \frac{3x^3 + 1}{x^4 - 5x^2 + 3}$$

$$D = \lim_{x \rightarrow \infty} \frac{x^2 + \sin(x)}{x^2 - \cos(x)}$$

Find $A + B + C + D$.

2. A = slope of the line normal to $x^3 y + y \sin(x) + y^3 = 8$ at the point $(0, 2)$.

$$B = \lim_{x \rightarrow \sqrt{\pi}} \frac{\sin(x^2)}{x - \sqrt{\pi}}$$

Find B^A .

3. A glass window is made by surmounting an equilateral triangle on top of a rectangle. If the area of the window is $6 - \sqrt{3}$ square feet, what is the minimum perimeter of the window in feet.

4. Below is a table containing various functional values for a twice differentiable function $f(x)$.

x	$f(x)$	$f'(x)$	$f''(x)$
1	1.5	40	-48
2	20	0	-30
3	9.5	-16	0
4	0	4	42

Write down the letter corresponding to each statement that is **NECESSARILY TRUE**.

- A. $f(x)$ has a root at $x = 4$
- B. $f(x)$ has a local maximum at $x = 2$
- C. $f(x)$ has a global maximum at $x = 2$
- D. $f(x)$ has a local minimum at $x = 4$
- E. $f(x)$ is decreasing at $x = 1$
- F. $f(x)$ has only one critical point
- G. There exists c such that $f(c) < 0$

5. Find all real values of a such that

$$\lim_{x \rightarrow a} \frac{x^2 + 2x + 1}{x - 3} = a.$$

6. A Major League baseball has a circumference of $9 \frac{1}{8}$ inches with a margin

of error of $\pm \frac{1}{8}$ inches. Find the relative error of the baseball's volume. Express your answer in cubic inches rounded to the nearest thousandth.

7. Consider the ellipse $2x^2 + y^2 = 1$. Suppose a triangle is inscribed in the ellipse such that one vertex is at $(0, -1)$ and the opposite side is parallel to the x -axis. What is the maximum area of such a triangle.

8. Below are six statements concerning an **odd, twice differentiable function** $f(x)$. If the statement is true, its value appears next to it in parentheses. If the statement is false, its value is 0. Find the sum of all the statements' values.

(-8) $f'(x)$ is odd

(15) $f''(x)$ is odd

(-12) $f'(x)$ is one-to-one

(41) $f''(x)$ is one-to-one

(-2) $f'(x)$ has a maximum value on $[-10, 10]$

(21) $f'(x)$ has a minimum value on $[-10, 10]$

9. $f(2) = 3$, $f'(2) = -2$, $f''(2) = 4$,

$g(2) = \ln 3$, $g'(2) = \frac{4}{3}$, and $g''(2) = -\frac{10}{9}$.

Find $D_x \left[g \left(\frac{1}{2} f(x) g'(x) \right) \right]$ at $x = 2$.

10. Two lampposts lie on opposite edges of a road which measures 20 feet across. Bob wishes run a wire underground to connect the two lampposts. It costs \$5 per foot if the wire runs underneath the road and \$3 per foot if the wire runs at the edge of the road. If the lampposts are 52 feet apart, what is the minimum cost of laying the wire

11. Find the point on the graph $y = x^2 + \frac{1}{2}$

that is closest to the point $\left(1, \frac{1}{4}\right)$.

12. Evaluate: $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\cos\left(\frac{\pi i}{2n}\right)}{n}$.

13. Suppose $f(x) = xe^x$.

$$A = f^{(2005)}(0)$$

$$B = f^{(2006)}(1)$$

$$C = f^{(2005)}(1)$$

$$D = f^{(2004)}(0)$$

Find $A + B - C - D$.

14. The line normal to the graph $y = x \cos(x)$ when $x = \pi$ is written in the form $y = Ax + B$.

Find $A \cdot B$.

15. Suppose a square initially has a side one inch long. The diagonal of a square is increasing at a rate of $\sqrt{6}$ inches per second. At what rate is the area of the square increasing when the side of the square is two inches long. Express your answer in square inches.