Calculus Team Question #1
January Regional 2003

Let $f(x) = (2x+1)^3 (x^2+2)^2 (x^5+3)$, let

$$A = f'(1) \qquad\qquad B = f'(0) \qquad\qquad C = f''(0)$$

Find $A + B - C$

Calculus Team Question #2
January Regional 2003

Given that $f(0) = 3$, $f'(0) = \dfrac{3}{5}$, $g(0) = 0$, and $g'(0) = 10$, let

$$A = f(x)g(x) \qquad\qquad B = f(g(x))$$

$$C = \frac{g(x)}{f(x)} \qquad\qquad D = (g(x))^4$$

Find $A'(0) \bullet B'(0) \bullet C'(0) \bullet D'(0)$

Calculus Team Question #3
January Regional 2003

Given $f(x) = x^3 + Ax^2 + Bx + C$ if $f(2) = f'(2) = f''(2) = 0$, find $A + B + C$

Calculus Team Question #4
January Regional 2003

Given $f(x) = 3x^2 - 12x$

A = Value of x guaranteed by Rolle's Theorem on [0,4]
B = Value of x guaranteed by the Mean Value Theorem on [0,2]
C = Average value of $f(x)$ on [-2,-1]
D = Minimum value of $f(x)$

Find $\dfrac{A+B}{C-D}$

Calculus Team Question #5
January Regional 2003

Let $f(x) = \begin{cases} ax^3 & 2 \le x \\ bx^2 + 4 & x < 2 \end{cases}$ Find the values of a and b so that $f(x)$ is both continuous and differentiable.

Given $f(x) = \int_0^{(x-1)^2} (4m - 5)dm$, let

$A = f(2)$, $B = f'(3)$, $C = f''(4)$, $D = f'''(5)$ Find $\dfrac{A \bullet C}{B \bullet D}$

Calculus Team Question #7
January Regional 2003

The side length of a cube is increasing at the rate of 3 units per minute.

Let A = the rate of change of the surface area of the cube.
Let B = the rate of change of the volume of the cube.
Let C = the rate of change of the length of a diagonal of the cube.

Ignoring the units, what is the exact value of A + B + C when the side length is 21 units?

Calculus Team Question #8
January Regional 2003

Find all vertical tangents for the relation $x^2 y + xy^2 + x = 0$

Calculus Team Question #9
January Regional 2003

The location of an object in motion is described by the equation

$x(t) = 3\sin(2t - \pi) - \dfrac{1}{2}t^2 + 3t + 4$, $t \geq 0$. Find the following values accurate to

three decimal places. Disregard units.

A) $v\left(\dfrac{\pi}{6}\right)$

B) $a\left(\dfrac{\pi}{6}\right)$

C) The value of $v(t)$ when $a(t) = 0$

D) The speed of the object when $t = \dfrac{5\pi}{4}$

Arrange these numeric values from largest to smallest.

Calculus Team Question #10
January Regional 2003

For $y = \sqrt{x^3}$, let $A = y'(125), B = y''(125), C = y'''(125)$. Find ABC as a fraction in lowest terms.

Calculus Team Question #11
January Regional 2003

$$A'(x) = \lim_{h \to 0} \frac{2^{(x+h)^2} - 2^{x^2}}{h}$$

$$C = A'(1)$$

$$B = A(1)$$

$$D = C(0)$$

What is the value of D?

Calculus Team Question #12
January Regional 2003

An element has a half-life of 2100 years. You have an original sample of 10 grams of the substance. In A years your sample is 8 grams. B years from that time your sample is 6 grams. Rounding both A and B to the nearest integer, find $A - B$.

Calculus Team Question #13
January Regional 2003

The rate of growth of a particular population is given by the function

$$\frac{dP}{dt} = 50t^2 - 100t^{3/2}$$ where P is the population size and t is the time in years. The

initial population is 25,000. In how many years (rounded to the nearest tenth of a year) will the population reach 50,000?

Calculus Team Question #14
January Regional 2003

Water is draining from a cone shaped filter at a constant rate of $3 cm^3/\text{sec}$. The cone is 60 centimeters high and 20 centimeters in diameter. When the cone is half full, how fast is the depth of the water changing? Express your answer accurate to three decimals.

Calculus Team Question #15
January Regional 2003

Let $f(x) = e^{\frac{-\sqrt{x}}{2}} + 3$ on the region $1 \le x \le 10$. If R represents the Riemann sum using the right hand rule, M represents the Riemann sum using the midpoint rule and L represents the Riemann sum using the left hand rule, rank them from smallest to largest.