Calculus Individual **FAMAT Regional** **January**

If none of the answers is correct choose e) NOTA.

1. Find the derivative of $\tan^3(5x)$.

a) $15\sec^2(5x)$ b) $15\sec^2(5x)\tan^2(5x)$ c) $15\sec^6(5x)$ d) $15\tan^2(5x)$ e) NOTA

2. $g'(x) = h(x)$, $f(x) = x^3$. $\frac{d}{dx}[f(g(x)] =$

a) $3x^2h(x)$ b) $3x^2g(x)$ c) $3g^2(x)h(x)$ d) $x^3h(x) + 3x^2g(x)$ e) NOTA

3. $g(x) = \lim_{h\to 0} \frac{2^{(x+h)^2} - 2^{x^2}}{h}$. Find g(1).

a) 4ln2 b) ln4 c) 4 d) ln8 e) NOTA

4. $f(x) = \frac{(x+m)(x-n)}{(x-r)^2}$. What are the equations of the asymptotes of the graph of f?

a) x = -m, x = n, x = r b) x = r, y = -mn c) x = r, y = 1 d) $x = r^2$, y = 11 e) NOTA

5. $y = \frac{5}{3+x^2}$. $\frac{dy}{dx} =$

a) $\frac{-10x}{3+x^2}$ b) $\frac{5}{2x}$ c) $\frac{-5x}{(3+x^2)^2}$ d) $\frac{-5}{(3+x^2)^2}$ e) NOTA

6. The line 2x + y - 3 = 0 is normal to the curve $y = x^4 + k$. Find the tenths digit of sin(k).

a) 5 b) 6 c) 7 d) 9 e) NOTA

7. $f(x) = \ln(x) + \sin(x)$. Which of the following is true at $x = \frac{\pi}{3}$?

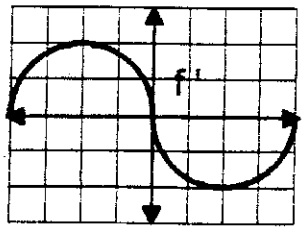
a) A relative maximum occurs at $x = \frac{\pi}{3}$. b) f is concave up.

c) f is increasing. d) $\lim_{x\to\frac{\pi}{3}} f'(x) = \frac{3}{\pi} - \frac{1}{2}$ e) NOTA

8. $x + xy + 2y^2 = 6$. What is the slope of the line tangent to the curve at (2,1)?

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{-1}{3}$ d) $\frac{-1}{5}$ e) NOTA

9. The graph of f ' over [-4,4] is given. Each tick mark represents a unit of one. Which of the following statements are true?



i) f has a max at x = -2
ii) The tangent line at x = 2 is parallel to 2x + y = 3.
iii) f is concave up over (-4, -2) and (2, 4).
iv) f is decreasing over (0, 4).

a) i, ii, iv b) ii, iii, iv c) ii, iii d) i, ii, iii, iv e) NOTA

10. The radius of a circle is increasing at a rate of k, k > 0, At a certain moment the rate of increase in the area of the circle is numerically twice the rate of increase in the circumference. What is the radius of the circle at this time?

a) $\frac{2}{k}$ b) $\frac{2}{\pi}$ c) 1 d) 2 e) NOTA

11. $f(x) = \sqrt{2x+1}$. $f'(4) =$

a) $\frac{1}{6}$ b) $\frac{1}{3}$ c) $\frac{9}{2}$ d) 9 e) NOTA

12. Use the linearization of f at $x = \frac{\pi}{4}$ to estimate f(0.7) to the nearest thousandth. $f(x) = \ln(\sec x)$.

a) 0.226 b) 0.261 c) 0.265 d) 0.271 e) NOTA

13. If $f(x) = \cos\left(\frac{x}{2}\right)$ in the interval $0 \le x \le \pi$, to the nearest thousandth which of the following could be the number c satisfying the conclusion of the Mean Value Theorem?

a) 0.345 b) 0.690 c) 1.380 d) 1.761 e) NOTA

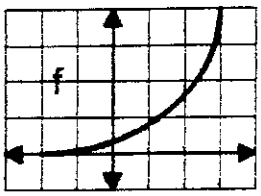
14. $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$. What value of k makes f continuous?

a) -3 b) 3 c) 6 d) 9 e) NOTA

15. $f'(x) = (x-3)^3(x-2)^2$. Which of the following statements is false?

a) f has critical values at x = 3 and x = 2.
b) f is increasing over $(3, \infty)$.
c) f has a minimum value occurring at x = 3.
d) f has two extreme values.
e) NOTA

16. $f(x) = gx^2 + hx + k$. Part of the graph of the parabola f is shown to the right. Each tick mark represents a unit of one. Which of the following must be true?



a) $g > 0, h > 0, k > 0$ b) $g < 0, h > 0, k > 0$
c) $g > 0, h < 0, k > 0$ d) $g > 0, h > 0, k < 0$ e) NOTA

17. The amount of water in gallons in a tank after a drainage pipe is opened is given by $W(t) = 300(20 - t)^2$, where t is the number of minutes since the pipe is first opened. What is the average rate of change in gallons per minute during the first 6 minutes?

a) -8, 400 b) -10,200 c) -61,200 d) -87,600 e) NOTA

18. Use differentials to approximate $\sqrt{67}$ using the fact that $\sqrt{64} = 8$. Round your approximation to the nearest thousandth.

a) 8.185 b) 8.186 c) 8.188 d) 8.191 e) NOTA

19. A company has a daily fixed cost of \$5000. If the company produces x units daily, then the daily cost in dollars for labor and materials is 3x. The daily cost of equipment maintenance is $\frac{x^2}{2{,}500{,}000}$. To the nearest unit what daily production rate minimizes the total daily cost per unit of production?

a) 50,000 b) 110,902 c) 111,803 d) 112,011 e) NOTA

20. Let $f(x) = [x]$, where [ ] denotes the greatest integer value of x. Which of the following statements are true for each integer J?

i) $\lim_{x \to J^-} [x] = J - 1$ ii) $\lim_{x \to J} [x] = J$ iii) For all x, $x \neq J$, f is continuous at x.

a) i b) i, ii c) i, iii d) i, ii, iii e) NOTA

21. What is the equation of the tangent line to $y = \arctan(3x)$ at $x = 0$.

a) $3x - y = 0$ b) $x - y = 0$ c) $x - 3y = 0$ d) $3x + y = 0$ e) NOTA

22. $x(t) = 2\sin(t) - 3\cos(t), t \geq 0$. x is the position function of a particle. To the nearest thousandth find the velocity of the particle the first time the acceleration is 0.

a) -1.387 b) 0.982 c) 3.328 d) 3.606 e) NOTA

23. $f(x) = e^{\frac{1}{x}}$. Which of the following statements is false?

a) $\lim_{x \to 0^+} f(x) = \infty$ b) f has no maxima of minima on its domain.
c) f decreases on its domain. d) f has no inflection points. e) NOTA

24. $f(0) = 2$ and f' is a constant function such that $3 \le f'(x) \le 7$ for all x in $[0,4]$. Which of the following must be true?

a) f has a critical value in $[0,4]$. b) $14 \le f(4) \le 30$
c) f is always concave up over $[0,4]$. d) f is not continuous at $x = 2$. e) NOTA

25. There exists a number a such that $\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$ exists. Find the tenths digit of ln a.

a) 1 b) 3 c) 6 d) 7 e) NOTA

26. A car is traveling at night along a highway shaped like a parabola with its vertex at the origin. The car starts at a point 100 m west and 100 m north of the origin and travels towards the origin. There is a statue located 100 m east and 50 m north of the origin. To the nearest thousandth what is the x-coordinate of the point on the highway when the car's headlights first illuminate the statue after passing through the origin?

a) 23.137 b) 29.289 c) 31.753 d) 35.265 e) NOTA

27. Which of the following is the y intercept of the line through the point (3,5) that cuts off the triangle of least area in the first quadrant?

a) 7 b) $\frac{43}{5}$ c) $\frac{49}{5}$ d) 10 e) NOTA

28. A water-skier skis over a ramp in the shape of a right triangle whose base is 15 ft and height is 4 ft. Her speed is a constant 30 ft/s. To the nearest thousandth in ft/s how fast is she rising as she leaves the ramp?

a) 7.730 b) 8.821 c) 12.400 d) 15.000 e) NOTA

29. Which of the following expressions does not have the same derivative as $y = \log_5(x)$?

a) $25 + \log_5(x)$ b) $\frac{1}{2}\log_{25}(x)$ c) $\log_5(8x)$ d) $\frac{\ln(x)}{\ln 5}$ e) NOTA

30. Let $f(x) = x^2 + \sqrt[3]{x-2}$. Which of the following describes the behavior of f at $x = 2$?

a) differentiable b) discontinuity c) cusp d) vertical tangent e) NOTA