

Factors and Powers

- 1 If x represents the number of positive integral factors of 2002, then find the largest prime number that is less than x .
A) 7 B) 11 C) 13
D) 17 E) NOTA

- 2 Find the sum, in base 6, of $2003_4 + 2004_5$.
A) 1441_6 B) 4001_6 C) 333_6
D) 225_6 E) NOTA

- 3 If the units digit in $2003^{227} = \textit{now}$, and the hundreds digit in $1980^{725} = \textit{then}$, find:
then – now
A) -7 B) 0 C) 1
D) 9 E) NOTA

- 4 Find the number of positive integral factors of 5544.
A) 12 B) 24 C) 48
D) 96 E) NOTA

- 5 How many digits are in the product of $(2002)^{2003} (2004)^{2005}$?
A) 13228 B) 13234 C) 30458
D) 30471 E) NOTA

- 6 Convert 123_4 to base 10
A) 113 B) 108 C) 27
D) 22 E) NOTA

- 7 Change 324 to an equivalent base 5 number
A) 324 B) 200 C) 2244
D) 2424 E) NOTA

- 8 Find the sum of all prime divisors of 1988
A) 80 B) 82 C) 83
D) 501 E) NOTA

- 9 The number 224_5 , written in base 5, is equal to KAT_n , written in base n . If the product $K \cdot A \cdot T = 0$, then n could be which of the following? (K, A , and T) are whole numbers < 10 and not necessarily distinct)
- A) 3 B) 6 C) 7
D) 8 E) NOTA
- 10 Find the sum of all integral powers of 2 between $\frac{1}{4}$ and 128, inclusive.
- A) $\frac{255}{2}$ B) 255 C) $\frac{1019}{4}$
D) $\frac{1023}{4}$ E) NOTA
- 11 Change 324 to an equivalent base 5 number.
- A) 324 B) 2200 C) 2244
D) 2424 E) NOTA
- 12 Find the sum of all prime divisors of 1988
- A) 80 B) 82 C) 83
D) 501 E) NOTA
- 13 When the decimal number 25^{52} is written in base 12, what is the units digit?
- A) 1 B) 5 C) 7
D) 9 E) NOTA
- 14 What is the units digit of $825^{824} - 827^{824} - 823^{824}$?
- A) 2 B) 3 C) 5
D) 6 E) NOTA
- 15 How many zeros are at the end of $32!$?
- A) 3 B) 6 C) 7
D) 9 E) NOTA
- 16 How many zeros are at the end of $345!$
- A) 69 B) 82 C) 84
D) 727 E) NOTA

- 17 If the units digit of 2^{2002} is x and the units digit of 3^{2002} is y , find the product of x and y .
- A) 4 B) 9 C) 13
D) 36 E) NOTA
- 18 Find the smallest natural number n such that $n!$ is divisible by 780
- A) 39 B) 19 C) 13
D) 10 E) NOTA
- 19 Find the units digit for the number $12^{21} + 64^{13} + 75^{19} + 81^{16}$
- A) 2 B) 4 C) 5
D) 1 E) NOTA
- 20 What is the ten's digit for $1! + 2! + 3! + 4! + \dots + 2002!$?
- A) 1 B) 2 C) 3
D) 4 E) NOTA
- 21 How many factors (natural numbers) does the number 12,600 have?
- A) 72 B) 120 C) 144
D) 56 E) NOTA

Solutions

13. $25 \equiv 1 \pmod{12}$; $25^{52} \equiv 1^{52} \equiv 1 \pmod{12}$

14. Units digit of $825^n = 5$ if $n > 0$

Units digit of $827^n = 7, 9, 3, 1$ based on n 's remainder when divided by 4.

Since the remainder in this case is zero, units digit = 1

Unit's digit of $823^n = 3, 9, 7, 1$ based on n 's remainder when divided by 4.

Since the remainder in this case is zero, units digit = 1 $5-1-1=3$